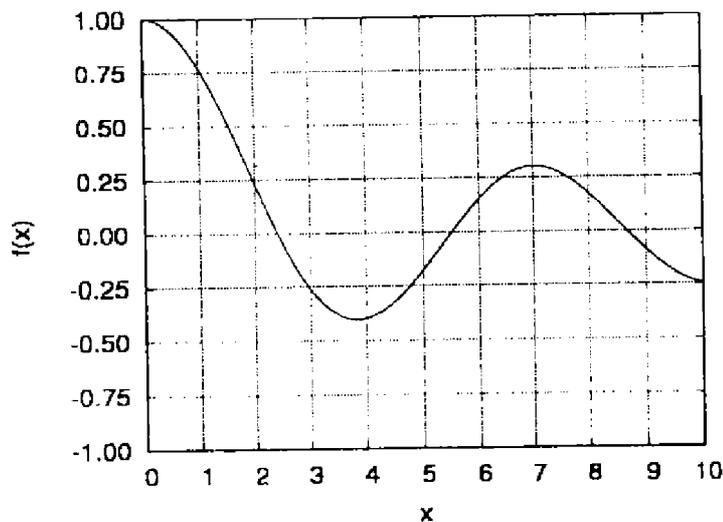


**CSIR-UGC-NET/JRF- DEC. - 2012**  
**PHYSICAL SCIENCES BOOKLET - [A]**

**Part-B**

21. A  $2 \times 2$  matrix  $A$  has eigenvalues  $e^{i\pi/5}$  and  $e^{i\pi/6}$ . The smallest value of 'n' such that  $A^n = 1$  is:  
 (a) 20                      (b) 30                      (c) 60                      (d) 120

22. The graph of the function  $f(x)$  as shown below is best described by



- (a) The bessel function  $J_0(x)$                       (b)  $\cos x$   
 (b)  $e^{-x} \cos x$                       (d)  $\frac{1}{x} \cos x$
23. In a series of five cricket matches, one of the captains calls "Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is  
 (a)  $1/8$                       (b)  $5/8$                       (c)  $3/16$                       (d)  $5/16$

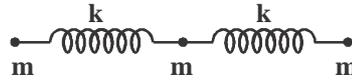
24. The unit normal vector at the point  $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$  on the surface of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \text{ is}$$

- (a)  $\frac{bc\hat{i} + ca\hat{j} + ab\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$       (b)  $\frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$       (c)  $\frac{b\hat{i} + c\hat{j} + a\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$       (d)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$
25. A solid cylinder of height  $H$ , radius  $R$  and density  $\rho$ , floats vertically on the surface of a liquid of density  $\rho_0$ . The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is

- (a)  $\frac{\rho g}{\rho_0 H}$                       (b)  $\frac{\rho}{\rho_0} \sqrt{\frac{g}{H}}$                       (c)  $\sqrt{\frac{\rho g}{\rho_0 H}}$                       (d)  $\sqrt{\frac{\rho_0 g}{\rho H}}$

26. Three particles of equal mass 'm' are connected by two identical massless springs of stiffness constant 'k' as shown in the figure.



If  $x_1$ ,  $x_2$  and  $x_3$  denote the displacements of the masses from their respective equilibrium positions, the potential energy of the system is:

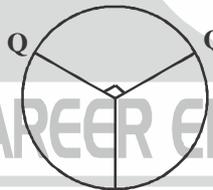
- (a)  $\frac{1}{2}k(x_1^2 + x_2^2 + x_3^2)$  (b)  $\frac{1}{2}k[x_1^2 + x_2^2 + x_3^2 - x_2(x_1 + x_3)]$   
 (c)  $\frac{1}{2}k[x_1^2 + 2x_2^2 + x_3^2 + 2x_2(x_1 + x_3)]$  (d)  $\frac{1}{2}k[x_1^2 + 2x_2^2 + x_3^2 - 2x_2(x_1 + x_3)]$
27. Let  $v$ ,  $p$  and  $E$  denotes the speed, the magnitude of the momentum, and the energy of a free particle of rest mass 'm'. Then

- (a)  $\frac{dE}{dp} = \text{constant}$  (b)  $p = mv$  (c)  $v = \frac{cp}{\sqrt{p^2 + m^2c^2}}$  (d)  $E = mc^2$

28. A binary star system consists of two stars  $S_1$  and  $S_2$ , with masses  $m$  and  $2m$  respectively separated by a distance 'r'. If both  $S_1$  and  $S_2$  individually follow circular orbits around the centre of the mass with instantaneous speeds  $v_1$  and  $v_2$  respectively, the ratio of speeds  $v_1/v_2$  is:

- (a)  $\sqrt{2}$  (b) 1 (c) 1/2 (d) 2

29. Three charges are located on the circumference of a circle of radius 'R' as shown in the figure below. The two charges  $Q$  subtend an angle  $90^\circ$  at the centre of the circle. The charge 'q' is symmetrically placed with respect to the charges  $Q$ . If the electric field at the centre of the circle is zero, what is the magnitude of  $Q$ ?

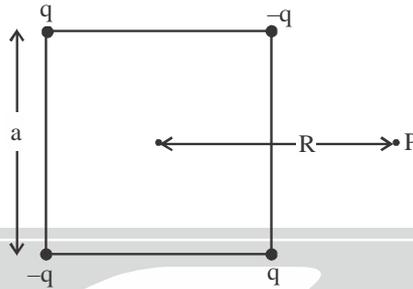


- (a)  $q/\sqrt{2}$  (b)  $\sqrt{2}q$  (c)  $2q$  (d)  $4q$

30. Consider a hollow charged shell of inner radius 'a' and outer radius 'b'. The volume charge density is  $\rho(r) = \frac{k}{r^2}$  (where  $k$  is a constant) in the region  $a < r < b$ . The magnitude of the electric field produced at distance  $r > a$  is:

- (a)  $\frac{k(b-a)}{\epsilon_0 r^2}$  for  $r > a$  (b)  $\frac{k(b-a)}{\epsilon_0 r^2}$  for  $a < r < b$  and  $\frac{kb}{\epsilon_0 r^2}$  for  $r > b$   
 (c)  $\frac{k(r-a)}{\epsilon_0 r^2}$  for  $a < r < b$  and  $\frac{k(b-a)}{\epsilon_0 r^2}$  for  $r > b$  (d)  $\frac{k(r-a)}{\epsilon_0 a^2}$  for  $a < r < b$  and  $\frac{k(b-a)}{\epsilon_0 a^2}$  for  $r > b$

31. Consider the interference of two coherent electromagnetic waves whose electric field vectors are given by  $\vec{E}_1 = \hat{i}E_0 \cos \omega t$  and  $\vec{E}_2 = \hat{j}E_0 \cos(\omega t + \varphi)$  where  $\varphi$  is the phase difference. The intensity of the resulting wave is given by  $\frac{\epsilon_0}{2} \langle E^2 \rangle$ , where  $\langle E^2 \rangle$  is the time average of  $E^2$ . The total intensity is
- (a) 0                      (b)  $\epsilon_0 E_0^2$                       (c)  $\epsilon_0 E_0^2 \sin^2 \varphi$                       (d)  $\epsilon_0 E_0^2 \cos^2 \varphi$
32. Four charges (two  $+q$  and two  $-q$ ) are kept fixed at the four vertices of a square of side 'a' as shown



At the point  $P$  which is at a distance  $R$  from the centre ( $R \gg a$ ), the potential is proportional to

- (a)  $\frac{1}{R}$                       (b)  $\frac{1}{R^2}$                       (c)  $\frac{1}{R^3}$                       (d)  $\frac{1}{R^4}$
33. A point charge ' $q$ ' of mass ' $m$ ' is kept at a distance ' $d$ ' below a grounded infinite conducting sheet which lies in the  $xy$ -plane. What is the value of ' $d$ ' for which the charge remains stationary?
- (a)  $q/4\sqrt{mg\pi\epsilon_0}$                       (b)  $q/\sqrt{mg\pi\epsilon_0}$   
 (c) There is no finite value of ' $d$ '                      (d)  $\sqrt{mg\pi\epsilon_0}/q$
34. The wave function of a state of the hydrogen atom is given by

$$\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\psi_{21-1}$$

where  $\psi_{n\ell m}$  denotes the normalized eigen function of the state with quantum numbers  $n$ ,  $\ell$  and  $m$  in the usual notation. The expectation value of  $L_z$  in the state  $\psi$  is:

- (a)  $\frac{15\hbar}{16}$                       (b)  $\frac{11\hbar}{16}$                       (c)  $\frac{3\hbar}{8}$                       (d)  $\frac{\hbar}{8}$
35. The energy eigenvalues of a particle in the potential  $V(x) = \frac{1}{2}m\omega^2 x^2 - ax$  are

- (a)  $E_n = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{a^2}{2m\omega^2}$                       (b)  $E_n = \left(n + \frac{1}{2}\right)\hbar\omega + \frac{a^2}{2m\omega^2}$   
 (c)  $E_n = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{a^2}{m\omega^2}$                       (d)  $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

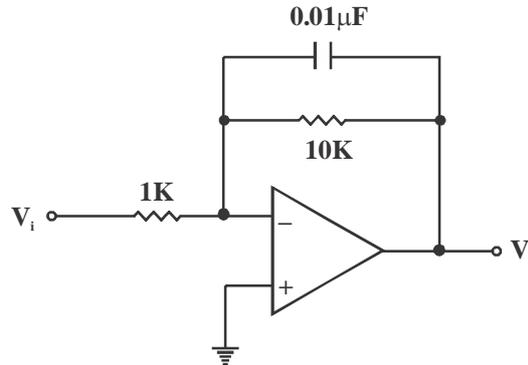
36. If a particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \frac{\sqrt{15}(a^2 - x^2)}{4a^{5/2}} & \text{for } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the uncertainty  $\Delta p$  in its momentum is

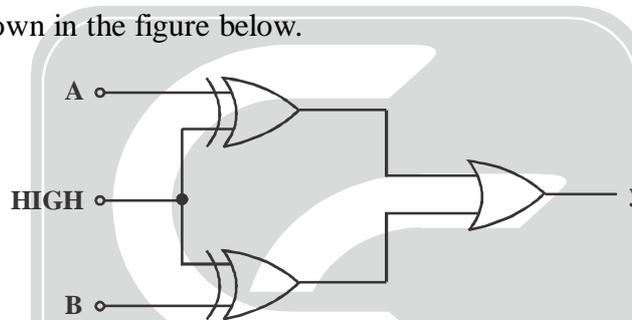
- (a)  $\frac{2\hbar}{5a}$       (b)  $\frac{5\hbar}{2a}$       (c)  $\frac{\sqrt{10}\hbar}{a}$       (d)  $\frac{\sqrt{5}\hbar}{\sqrt{2}a}$
37. Given the usual canonical commutation relations, the commutator  $[A, B]$  of  $A = i(xp_y - yp_x)$  and  $B = (yp_z + zp_y)$  is:
- (a)  $\hbar(xp_z - p_xz)$       (b)  $-\hbar(xp_z - p_xz)$       (c)  $\hbar(xp_z + p_xz)$       (d)  $-\hbar(xp_z + p_xz)$
38. The entropy of a system,  $S$ , is related to the accessible phase space volume  $\Gamma$  by  $S = k_B \ln \Gamma(E, N, V)$  where  $E$ ,  $N$  and  $V$  are the energy, number of particles and volume respectively. From this one can conclude that  $\Gamma$
- (a) does not change during evolution to equilibrium  
 (b) Oscillates during evolution to equilibrium  
 (c) Is a maximum in equilibrium  
 (d) Is a minimum in equilibrium
39. Let  $\Delta W$  be the work done in a quasistatic reversible thermodynamics process. Which of the following statements about  $\Delta W$  is correct?
- (a)  $\Delta W$  is a perfect differential if the process is isothermal  
 (b)  $\Delta W$  is a perfect differential if the process is adiabatic  
 (c)  $\Delta W$  is always a perfect differential  
 (d)  $\Delta W$  cannot be a perfect differential.
40. Consider a system of three spins  $S_1, S_2$  and  $S_3$  each of which can take values  $+1$  and  $-1$ . The energy of the system is given by  $E = -J[S_1S_2 + S_2S_3 + S_3S_1]$ , where  $J$  is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively,
- (a)  $J$  and 1      (b)  $-3J$  and 1      (c)  $-3J$  and 2      (d)  $-6J$  and 2
41. The minimum energy of a collection of 6 non-interacting electrons of spin  $-\frac{1}{2}$  placed in a one dimensional infinite square well potential of width  $L$  is
- (a)  $14\pi^2\hbar^2 / mL^2$       (b)  $9\pi^2\hbar^2 / mL^2$       (c)  $7\pi^2\hbar^2 / mL^2$       (d)  $3\pi^2\hbar^2 / mL^2$
42. A live music broadcast consists of a radio-wave of frequency 7 MHz, amplitude-modulated by a microphone output consisting of signals with a maximum frequency of 10 KHz. The spectrum of modulated output will be zero outside the frequency band
- (a) 7.00 MHz to 7.01 MHz      (b) 6.99 MHz to 7.01 MHz  
 (c) 6.99 MHz to 7.00 MHz      (d) 6.995 MHz to 7.005 MHz

43. In the op-amp circuit shown in the figure,  $V_i$  is a sinusoidal input signal of frequency 10 Hz and  $V_o$  is the output signal.



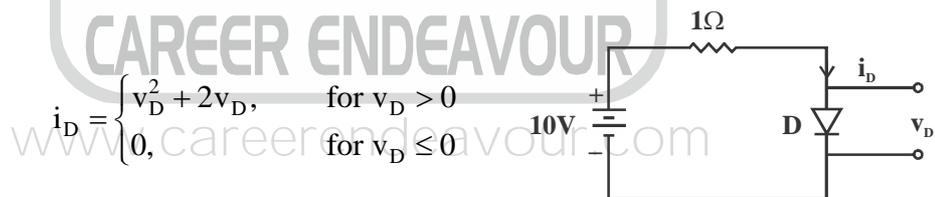
The magnitude of the gain and the phase shift, respectively, are close to the values

- (a)  $5\sqrt{2}$  and  $\frac{\pi}{2}$       (b)  $5\sqrt{2}$  and  $-\frac{\pi}{2}$       (c) 10 and zero      (d) 10 and  $\pi$
44. The logic circuit shown in the figure below.



implements the Boolean expression

- (a)  $y = \overline{A.B}$       (b)  $y = \overline{A.B}$       (c)  $y = A.B$       (d)  $y = A + B$
45. A diode D as shown in the circuit as an  $i-v$  relation which can be proximated by



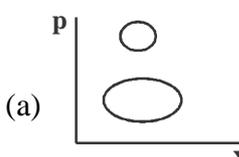
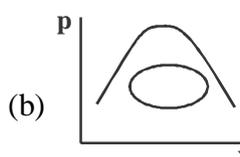
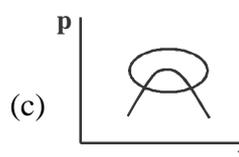
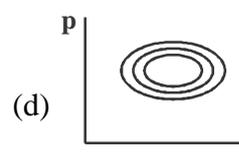
The value of  $v_D$  in the circuit is:

- (a)  $(-1 + \sqrt{11})V$       (b) 8V      (c) 5V      (d) 2V

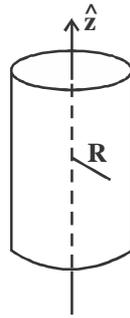
**Part-C**

46. The Taylor expansion of the function  $\ln(\cosh x)$ , where 'x' is real, about the point  $x = 0$  starts with the following terms:

- (a)  $-\frac{1}{2}x^2 + \frac{1}{12}x^4 + \dots$       (b)  $\frac{1}{2}x^2 - \frac{1}{12}x^4 + \dots$   
 (c)  $-\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$       (d)  $\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$

47. Given a  $2 \times 2$  unitary matrix  $U$  satisfying  $U'U = UU' = 1$  with  $\det U = e^{i\phi}$ , one can construct a unitary matrix  $V$  ( $V'V = VV' = 1$ ) with  $\det V = 1$  from it by
- (a) Multiplying  $U$  by  $e^{-i\phi/2}$  (b) Multiplying any single element of  $U$  by  $e^{-i\phi}$   
 (c) Multiplying any row or column of  $U$  by  $e^{-i\phi/2}$  (d) Multiplying  $U$  by  $e^{-i\phi}$ .
48. The value of the integral  $\int_C \frac{z^3 dz}{z^2 - 5z + 6}$ , where  $C$  is a closed contour defined by the equation  $2|z| - 5 = 0$ , traversed in the anti-clockwise direction, is:
- (a)  $-16\pi i$  (b)  $16\pi i$  (c)  $8\pi i$  (d)  $2\pi i$
49. A function  $f(x)$  obeys the differential equation  $\frac{d^2 f}{dx^2} - (3 - 2i)f = 0$  and satisfies the conditions  $f(0) = 1$  and  $f(x) \rightarrow 0$  as  $x \rightarrow \infty$ . The value of  $f(\pi)$  is:
- (a)  $e^{2\pi}$  (b)  $e^{-2\pi}$  (c)  $-e^{-2\pi}$  (d)  $-e^{2\pi i}$
50. A planet of mass 'm' moves in the gravitational field of the Sun (mass  $M$ ). If the semi-major and semi-minor axes of the orbit are 'a' and 'b' respectively, the angular momentum of the planet is:
- (a)  $\sqrt{2GMm^2(a+b)}$  (b)  $\sqrt{2GMm^2(a-b)}$   
 (c)  $\sqrt{\frac{2GMm^2 ab}{(a-b)}}$  (d)  $\sqrt{\frac{2GMm^2 ab}{(a+b)}}$
51. The Hamiltonian of a simple pendulum consisting of a mass 'm' attached to a massless string of length  $l$  is  $H = \frac{p_\theta^2}{2m\ell^2} + mg\ell(1 - \cos\theta)$ . If  $L$  denotes the Lagrangian, the value of  $\frac{dL}{dt}$  is:
- (a)  $-\frac{2g}{\ell} p_\theta \sin\theta$  (b)  $-\frac{g}{\ell} p_\theta \sin 2\theta$  (c)  $\frac{g}{\ell} p_\theta \cos\theta$  (d)  $\ell p_\theta^2 \cos\theta$
52. Which of the following set of phase-space trajectories which one is not possible for a particle obeying Hamilton's equations of motion (for a time-independent Hamiltonian)?
- (a)  (b)  (c)  (d) 
53. Two bodies of equal mass 'm' are connected by a massless rigid rod of length 'l' lying in the  $xy$ -plane with the centre of the rod at the origin. If this system is rotating about the  $z$ -axis with a frequency  $\omega$ , its angular momentum is
- (a)  $m\ell^2\omega/4$  (b)  $m\ell^2\omega/2$  (c)  $m\ell^2\omega$  (d)  $2m\ell^2\omega$

54. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current I.



The vector potential  $\vec{A}$  at a distance R from the axis.

- (a) Is constant inside and varies as R outside the solenoid.  
 (b) Varies as R inside and is constant outside the solenoid.  
 (c) Varies as  $1/R$  inside and as R outside the solenoid.  
 (d) Varies as R inside and as  $1/R$  outside the solenoid.
55. Consider an infinite conducting sheet in the xy-plane with a time dependent current density  $Kt\hat{i}$ , where K is a constant. The vector potential at (x, y, z) is given by

$$\vec{A} = \frac{\mu_0 K}{4c} (ct - z)^2 \hat{i}$$

The magnetic field  $\vec{B}$  is:

- (a)  $\frac{\mu_0 Kt}{2} \hat{j}$       (b)  $-\frac{\mu_0 Kz}{2c} \hat{j}$       (c)  $-\frac{\mu_0 K}{2c} (ct - z) \hat{i}$       (d)  $-\frac{\mu_0 K}{2c} (ct - z) \hat{j}$
56. When a charged particle emits electromagnetic radiation, the electric field  $\vec{E}$  and the Poynting vector  $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$  at a large distance 'r' from the emitter vary as  $\frac{1}{r^n}$  and  $\frac{1}{r^m}$  respectively. Which of the following choices for 'n' and 'm' are correct?  
 (a)  $n = 1$  and  $m = 1$       (b)  $n = 2$  and  $m = 2$       (c)  $n = 1$  and  $m = 2$       (d)  $n = 2$  and  $m = 4$

57. The energies in the ground state and first excited state of a particle of mass  $m = \frac{1}{2}$  in a potential  $V(x)$  are  $-4$  and  $-1$ , respectively, (in units in which  $\hbar = 1$ ). If the corresponding wavefunctions are related by  $\psi_1(x) = \psi_0(x) \sinh x$ , then the ground state eigenfunction is

- (a)  $\psi_0(x) = \sqrt{\sec hx}$       (b)  $\psi_0(x) = \sec hx$   
 (c)  $\psi_0(x) = \sec h^2 x$       (d)  $\psi_0(x) = \sec h^3 x$

58. The perturbation

$$H' = \begin{cases} b(a-x) & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

acts on a particle of mass 'm' confined in an infinite square well potential

$$V(x) = \begin{cases} 0 & -a < x < a \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to the ground state energy of the particle is

- (a)  $\frac{ba}{2}$                       (b)  $\frac{ba}{\sqrt{2}}$                       (c)  $2ba$                       (d)  $ba$

59. Let  $|0\rangle$  and  $|1\rangle$  denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty  $\Delta x$  in the state  $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$  is:

- (a)  $\Delta x = \sqrt{\hbar/2m\omega}$     (b)  $\Delta x = \sqrt{\hbar/m\omega}$     (c)  $\Delta x = \sqrt{2\hbar/m\omega}$     (d)  $\Delta x = \sqrt{\hbar/4m\omega}$

60. What would be the ground state energy of the Hamiltonian

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} - \alpha \delta(x)$$

if vibrational principle is used to estimate it with the trial wavefunction  $\psi(x) = A e^{-bx^2}$  with  $b$  as the variational parameter?

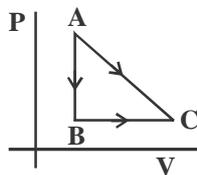
[Hint:  $\int_{-\infty}^{\infty} x^{2n} e^{-2bx^2} dx = (2b)^{-n-\frac{1}{2}} \Gamma\left(n + \frac{1}{2}\right)$ ]

- (a)  $-\frac{m\alpha^2}{2\hbar^2}$                       (b)  $-\frac{2m\alpha^2}{\pi\hbar^2}$                       (c)  $-\frac{m\alpha^2}{\pi\hbar^2}$                       (d)  $\frac{m\alpha^2}{\pi\hbar^2}$

61. The free energy difference between the superconducting and the normal states of a material is given by  $\Delta F = F_S - F_N = \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4$ , where  $\psi$  is an order parameter and  $\alpha$  and  $\beta$  are constants such that  $\alpha > 0$  in the normal and  $\alpha < 0$  in the superconducting state, while  $\beta > 0$  always. The minimum value of  $\Delta F$  in the superconducting state is

- (a)  $-\alpha^2/\beta$                       (b)  $-\alpha^2/2\beta$                       (c)  $-3\alpha^2/2\beta$                       (d)  $-5\alpha^2/2\beta$

62. A given quantity of gas is taken from the state  $A \rightarrow C$  reversibly, by two paths,  $A \rightarrow C$  directly and  $A \rightarrow B \rightarrow C$  as shown in the figure below.



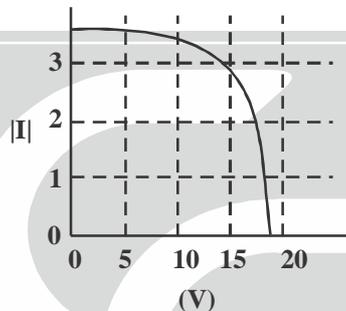
During the  $A \rightarrow C$  the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process  $A \rightarrow B \rightarrow C$  the work done by the gas is 30 J, the heat absorbed is:

- (a) 20J                      (b) 80J                      (c) 220 J                      (d) 280 J

63. Consider a one-dimensional Ising model with  $N$  spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy  $2J$ . In a configuration with  $r$  spin flips, the energy of the system is  $E = -NJ + 2rJ$  and the number of configuration is  ${}^N C_r$ ;  $r$  varies from 0 to  $N$ . The partition function is

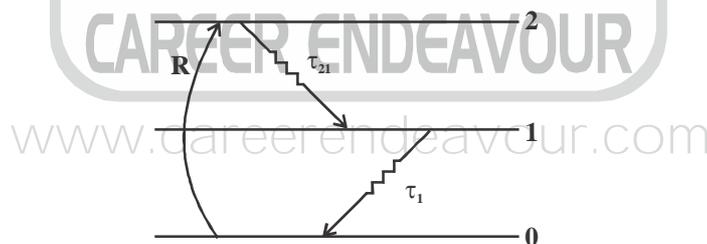
- (a)  $\left(\frac{J}{k_B T}\right)^N$                       (b)  $e^{-NJ/k_B T}$                       (c)  $\left(\sinh \frac{J}{k_B T}\right)^N$                       (d)  $\left(\cosh \frac{J}{k_B T}\right)^N$

64. A magnetic field sensor based on the Hall effect is to be fabricated by implanting. As into a Si film of thickness  $1 \mu\text{m}$ . The specifications require a magnetic field sensitivity of  $500 \text{ mV/Tesla}$  at an excitation current of  $1 \text{ mA}$ . The implantation dose is to be adjusted such that the average carrier density, after activation, is
- (a)  $1.25 \times 10^{26} \text{ m}^{-3}$     (b)  $1.25 \times 10^{22} \text{ m}^{-3}$     (c)  $4.1 \times 10^{21} \text{ m}^{-3}$     (d)  $4.1 \times 10^{20} \text{ m}^{-3}$
65. Band-pass and band-reject filters can be implemented by combining a low pass and a high pass filter in series and in parallel, respectively. If the cut-off frequencies of the low pass and high pass filters are  $\omega_0^{\text{LP}}$  and  $\omega_0^{\text{HP}}$ , respectively, the condition required to implement the band-pass and band-reject filters are respectively.
- (a)  $\omega_0^{\text{HP}} < \omega_0^{\text{LP}}$  and  $\omega_0^{\text{HP}} < \omega_0^{\text{LP}}$                       (b)  $\omega_0^{\text{HP}} < \omega_0^{\text{LP}}$  and  $\omega_0^{\text{HP}} > \omega_0^{\text{LP}}$
- (c)  $\omega_0^{\text{HP}} > \omega_0^{\text{LP}}$  and  $\omega_0^{\text{HP}} < \omega_0^{\text{LP}}$                       (d)  $\omega_0^{\text{HP}} > \omega_0^{\text{LP}}$  and  $\omega_0^{\text{HP}} > \omega_0^{\text{LP}}$
66. The output characteristics of a solar panel at a certain level of irradiance is shown in the figure below.



If the solar cell is to power a load of  $5 \Omega$ , the power drawn by the load is:

- (a) 97W                      (b) 73 W                      (c) 50 W                      (d) 45 W
67. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate  $R$  is  $10^{20} \text{ atoms cm}^{-3} \text{ s}^{-1}$  and the decay routes are as shown with  $\tau_{21} = 20 \text{ ns}$  and  $\tau_1 = 1 \mu\text{s}$ , the equilibrium populations of states 2 and 1 are, respectively,

- (a)  $10^{14} \text{ cm}^{-3}$  and  $2 \times 10^{12} \text{ cm}^{-3}$                       (b)  $2 \times 10^{12} \text{ cm}^{-3}$  and  $10^{14} \text{ cm}^{-3}$
- (c)  $2 \times 10^{12} \text{ cm}^{-3}$  and  $2 \times 10^6 \text{ cm}^{-3}$                       (d) zero and  $10^{20} \text{ cm}^{-3}$ .
68. Consider a hydrogen atom undergoing a  $2P \rightarrow 1S$  transition. The lifetime  $t_{\text{sp}}$  of the  $2P$  state for spontaneous emission is  $1.6 \text{ ns}$  and the energy difference between the levels is  $10.2 \text{ eV}$ . Assuming that the refractive index of the medium  $n_0 = 1$ , the ratio of the Einstein coefficients for stimulated emission  $B_{21}(\omega)/A_{21}(\omega)$  is given by
- (a)  $0.683 \times 10^{12} \text{ m}^3 \text{ J}^{-1} \text{ s}^{-1}$                       (b)  $0.146 \times 10^{-12} \text{ J s m}^{-3}$
- (c)  $6.83 \times 10^{12} \text{ m}^3 \text{ J}^{-1} \text{ s}^{-1}$                       (d)  $1.463 \times 10^{-12} \text{ J s m}^{-3}$



69. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities  $R_1 = 1$  and  $R_2 = 0.98$ . The mirrors are separated by a distance  $d = 20$  cm and the medium in between has a refractive index  $n_0 = 1$  and absorption coefficient  $\alpha = 0$ . The values of the separation between the modes  $\delta\nu$  and the width  $\Delta\nu_p$  of each mode of the laser cavity are:
- (a)  $\delta\nu = 75$  kHz,  $\Delta\nu_p = 24$  kHz                      (b)  $\delta\nu = 100$  kHz,  $\Delta\nu_p = 100$  kHz  
 (c)  $\delta\nu = 750$  MHz,  $\Delta\nu_p = 2.4$  MHz                      (d)  $\delta\nu = 2.4$  MHz,  $\Delta\nu_p = 750$  MHz
70. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to
- (a)  $\hbar\omega/2$                       (b)  $\hbar\omega$                       (c)  $3\hbar\omega/2$                       (d) 0
71. In a band structure calculation, the dispersion relation for electrons is found to be
- $$\varepsilon_k = \beta(\cos k_x a + \cos k_y a + \cos k_z a)$$
- where  $\beta$  is a constant and  $a$  is the lattice constant. The effective mass at the boundary of the first Brillouin zone is
- (a)  $\frac{2\hbar^2}{5\beta a^2}$                       (b)  $\frac{4\hbar^2}{5\beta a^2}$                       (c)  $\frac{\hbar^2}{2\beta a^2}$                       (d)  $\frac{\hbar^2}{3\beta a^2}$
72. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is  $a^3$ , is
- (a)  $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$                       (b)  $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$                       (c)  $\left(\frac{\pi^2}{a^3}\right)^{1/3}$                       (d)  $\frac{1}{a}$
73. The muon has mass  $105$  MeV/ $c^2$  and mean lifetime  $2.2$   $\mu$ s in its rest frame. The mean distance traversed by muon of energy  $315$  MeV/ $c^2$  before decaying is approximately
- (a)  $3 \times 10^5$  km                      (b) 2.2 cm                      (c) 6.6  $\mu$ m                      (d) 1.98 km
74. Consider the following particles: the proton  $p$ , the neutron  $n$ , the neutral pion  $\pi^0$  and the delta resonance  $\Delta^+$ . When ordered in terms of decreasing lifetime, the correct arrangement is as follows:
- (a)  $\pi^0, n, p, \Delta^+$                       (b)  $p, n, \Delta^+, \pi^0$                       (c)  $p, n, \pi^0, \Delta^+$                       (d)  $\Delta^+, n, \pi^0, p$
75. The single particle energy difference between the  $p$ -orbitals (i.e.  $p_{3/2}$  and  $p_{1/2}$ ) of the nucleus  ${}_{50}^{114}\text{Sn}$  is 3 MeV. The energy difference between the states in its  $1f$  orbital is
- (a)  $-7$  MeV                      (b) 7 MeV                      (c) 5 MeV                      (d)  $-5$  MeV