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PHYSICAL SCIENCES BOOKLET-[A]

PART-B

21. In the scattering of some elementary particles, the scattering cross-section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of σ on these quantities is given by

(1) $\sqrt{\frac{hc}{E}}$ (2) $\frac{hc}{E^{3/2}}$ (3) $\left(\frac{hc}{E}\right)^2$ (4) $\frac{hc}{E}$

22. If $y = \frac{1}{\tan h(x)}$, then x is

(1) $\ln\left(\frac{y+1}{y-1}\right)$ (2) $\ln\left(\frac{y-1}{y+1}\right)$ (3) $\ln\sqrt{\frac{y-1}{y+1}}$ (4) $\ln\sqrt{\frac{y+1}{y-1}}$

23. The function $\frac{z}{\sin \pi z^2}$ of a complex variable z has

- (1) a simple pole at 0 and poles of order 2 at $\pm\sqrt{n}$ for $n=1, 2, 3, \dots$
- (2) a simple pole at 0 and poles of order 2 at $\pm\sqrt{n}$ and $\pm i\sqrt{n}$ for $n=1, 2, 3, \dots$
- (3) poles of order 2 at $\pm\sqrt{n}$, $n=0, 1, 2, 3, \dots$
- (4) poles of order 2 at $\pm n$, $n=0, 1, 2, 3, \dots$

24. The Fourier transform of $f(x)$ is $\tilde{f}(k) = \int_{-\infty}^{+\infty} dx e^{ikx} f(x)$. If $f(x) = \alpha\delta(x) + \beta\delta'(x) + \gamma\delta''(x)$, where $\delta(x)$ is the Dirac delta-function (and prime denotes derivative), what is $\tilde{f}(k)$?

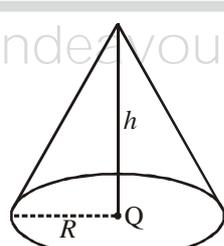
(1) $\alpha + i\beta k + i\gamma k^2$ (2) $\alpha + \beta k - \gamma k^2$
(3) $\alpha - i\beta k - \gamma k^2$ (4) $i\alpha + \beta k - i\gamma k^2$

25. The solution of the differential equation $\frac{dx}{dt} = 2\sqrt{1-x^2}$, with initial condition $x=0$ at $t=0$ is

(1) $x = \begin{cases} \sin 2t, & 0 \leq t < \frac{\pi}{4} \\ \sin h 2t, & t \geq \frac{\pi}{4} \end{cases}$ (2) $x = \begin{cases} \sin 2t, & 0 \leq t < \frac{\pi}{2} \\ 1, & t \geq \frac{\pi}{2} \end{cases}$
(3) $x = \begin{cases} \sin 2t, & 0 \leq t < \frac{\pi}{4} \\ 1, & t \geq \frac{\pi}{4} \end{cases}$ (4) $x = 1 - \cos 2t, t \geq 0$

26. A particle moves in three-dimensional space in a central potential $V(r) = kr^4$, where k is a constant. The angular frequency ω for a circular orbit depends on its radius R as

(1) $\omega \propto R$ (2) $\omega \propto R^{-1}$ (3) $\omega \propto R^{1/4}$ (4) $\omega \propto R^{-2/3}$

27. Two masses, m each, are placed at the points $(x, y) = (a, a)$ and $(-a, -a)$, and two masses, $2m$ each, are placed at the points $(a, -a)$ and $(-a, a)$. The principal moments of inertia of the system are
- (1) $2ma^2, 4ma^2$ (2) $4ma^2, 8ma^2$
 (3) $4ma^2, 4ma^2$ (4) $8ma^2, 8ma^2$
28. The Lagrangian of a system is given by $L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - k\left(\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2\right)$ where m and k are positive constants. The frequencies of its normal modes are
- (1) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{3k}{m}}$ (2) $\sqrt{\frac{k}{2m}}(13 \pm \sqrt{73})$
 (3) $\sqrt{\frac{5k}{2m}}, \sqrt{\frac{k}{m}}$ (4) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{6k}{m}}$
29. Consider a particle of mass m moving with a speed v . If T_R denotes the relativistic kinetic energy and T_N its non-relativistic approximation, then the value of $(T_R - T_N)/T_R$ for $v = 0.01c$, is
- (1) 1.25×10^{-5} (2) 5.0×10^{-5} (3) 7.5×10^{-5} (4) 1.0×10^{-4}
30. A hollow metallic sphere of radius a , which is kept at a potential V_0 , has a charge Q at its centre. The potential at a point outside the sphere, at a distance r from the centre, is
- (1) V_0 (2) $\frac{Q}{4\pi\epsilon_0 r} + \frac{V_0 a}{r}$ (3) $\frac{Q}{4\pi\epsilon_0 r} + \frac{V_0 a^2}{r^2}$ (4) $\frac{V_0 a}{r}$
31. Consider a charge Q at the origin of 3-dimensional coordinate system. The flux of the electric field through the curved surface of a cone that has a height h and a circular base of radius R (as shown in the figure) is
- (1) $\frac{Q}{\epsilon_0}$ (2) $\frac{Q}{2\epsilon_0}$
 (3) $\frac{hQ}{R\epsilon_0}$ (4) $\frac{QR}{2h\epsilon_0}$
- 
32. Given a uniform magnetic field $\mathbf{B} = B_0 \hat{k}$ (where B_0 is a constant), a possible choice for the magnetic vector potential \mathbf{A} is
- (1) $B_0 y \hat{i}$ (2) $-B_0 y \hat{i}$ (3) $B_0(x \hat{j} + y \hat{i})$ (4) $B_0(x \hat{i} - y \hat{j})$
33. A beam of unpolarized light in a medium with dielectric constant ϵ_1 is reflected from a plane interface formed with another medium of dielectric constant $\epsilon_2 = 3\epsilon_1$. The two media have identical magnetic permeability. If the angle of incidence is 60° , then the reflected light
- (1) is plane polarized perpendicular to the plane of incidence

39. For a system of independent non-interacting one-dimensional oscillators, the value of the free energy per oscillator, in the limit $T \rightarrow 0$, is

- (1) $\frac{1}{2}\hbar\omega$ (2) $\hbar\omega$ (3) $\frac{3}{2}\hbar\omega$ (4) 0

40. The partition function of a system of N Ising spins is $Z = \lambda_1^N + \lambda_2^N$, where λ_1 and λ_2 are functions of temperature, but are independent of N . If $\lambda_1 > \lambda_2$, the free energy per spin in the limit $N \rightarrow \infty$ is

- (1) $-k_B T \ln\left(\frac{\lambda_1}{\lambda_2}\right)$ (2) $-k_B T \ln \lambda_2$ (3) $-k_B T \ln(\lambda_1 \lambda_2)$ (4) $-k_B T \ln \lambda_1$

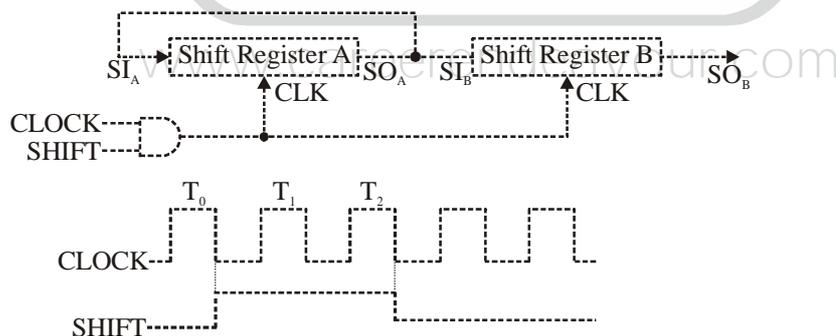
41. The Hamiltonian of a system of N non-interacting spin- $\frac{1}{2}$ particles is $H = -\mu_0 B \sum_i S_i^z$, where $S_i^z = \pm 1$ are the components of i^{th} spin along an external magnetic field B . At a temperature T such that $e^{\mu_0 B/k_B T} = 2$, the specific heat per particle is

- (1) $\frac{16}{25}k_B$ (2) $\frac{8}{25}k_B \ln 2$ (3) $k_B (\ln 2)^2$ (4) $\frac{16}{25}k_B (\ln 2)^2$

42. If the reverse bias voltage of a silicon varactor is increased by a factor of 2, the corresponding transition capacitance

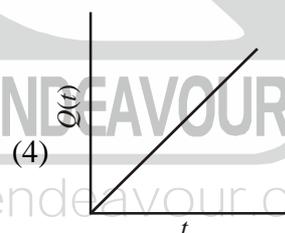
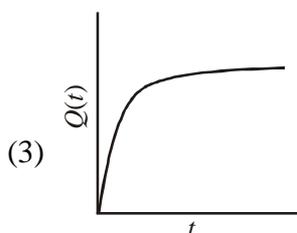
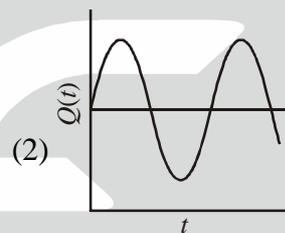
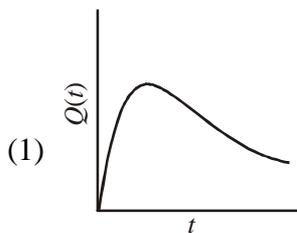
- (1) increases by a factor of $\sqrt{2}$ (2) increases by a factor of 2
(3) decreases by a factor of $\sqrt{2}$ (4) decreases by a factor of 2

43. In the schematic figure given below, the initial values of 4 bit shift registers A and B are 1011 and 0010 respectively. The values of SO_A and SO_B after the pulse T_2 are respectively.

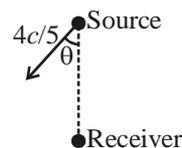


- (1) 1110 and 1001 (2) 1101 and 1001
(3) 1101 and 1100 (4) 1110 and 1100

48. For a dynamical system governed by the equation $\frac{dx}{dt} = 2\sqrt{1-x^2}$, with $|x| \leq 1$,
- (1) $x = -1$ and $x = 1$ are both unstable fixed points
 - (2) $x = -1$ and $x = 1$ are both stable fixed points
 - (3) $x = -1$ is an unstable fixed point and $x = 1$ is a stable fixed point
 - (4) $x = -1$ is stable fixed point and $x = 1$ is an unstable fixed point
49. The value of the integral $\int_0^8 \frac{1}{x^2+5} dx$, evaluated using Simpson's $\frac{1}{3}$ rule with $h = 2$, is
- (1) 0.565
 - (2) 0.620
 - (3) 0.698
 - (4) 0.736
50. A canonical transformation $(p, q) \rightarrow (P, Q)$ is performed on the Hamiltonian $H = \frac{1}{2m} p^2 + \frac{1}{2} m \omega^2 q^2$ via the generating function $F = \frac{1}{2} m \omega q^2 \cot Q$. If $Q(0) = 0$, which of the following graphs shows schematically the dependence of $Q(t)$ on t ?



51. A distant source, emitting radiation of frequency ω , moves with a velocity $4c/5$ in a certain direction with respect to a receiver (as shown in the figure).



The upper cut-off frequency of the receiver is $3\omega/2$. Let θ be the angle as shown. For the receiver to detect the radiation, θ should at least be

- (1) $\cos^{-1}\left(\frac{1}{2}\right)$
- (2) $\cos^{-1}\left(\frac{3}{4}\right)$
- (3) $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$
- (4) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

52. The Lagrangian of a particle moving in a plane is given in Cartesian coordinates as $L = \dot{x} \dot{y} - x^2 - y^2$. In polar coordinates the expression for the canonical momentum p_r (conjugate to the radial coordinate r) is
- (1) $\dot{r} \sin \theta + r \dot{\theta} \cos \theta$ (2) $\dot{r} \cos \theta + r \dot{\theta} \sin \theta$
 (3) $2\dot{r} \cos 2\theta - r \dot{\theta} \sin 2\theta$ (4) $\dot{r} \sin 2\theta + r \dot{\theta} \sin 2\theta$
53. A small magnetic needle is kept at (0, 0) with its moment along the x -axis. Another small magnetic needle is at the point (1, 1) and is free to rotate in the xy -plane. In equilibrium the angle θ between their magnetic moments is such that
- (1) $\tan \theta = 1/3$ (2) $\tan \theta = 0$
 (3) $\tan \theta = 3$ (4) $\tan \theta = 1$
54. A dipole of moment \vec{p} , oscillating at frequency ω , radiates spherical waves. The vector potential at large distance is $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r} \vec{p}$. To order $(1/r)$ the magnetic field \vec{B} at a point $\vec{r} = r\hat{n}$ is
- (1) $-\frac{\mu_0}{4\pi} \frac{\omega^2}{c} (\hat{n} \cdot \vec{p}) \hat{n} \frac{e^{ikr}}{r}$ (2) $-\frac{\mu_0}{4\pi} \frac{\omega^2}{c} (\hat{n} \times \vec{p}) \frac{e^{ikr}}{r}$
 (3) $-\frac{\mu_0}{4\pi} \omega^2 k (\hat{n} \cdot \vec{p}) \vec{p} \frac{e^{ikr}}{r}$ (4) $-\frac{\pi_0}{4\pi} \frac{\omega}{c} \vec{p} \frac{e^{ikr}}{r}$
55. The frequency dependent dielectric constant of a material is given by $\epsilon(\omega) = 1 + \frac{A}{\omega_0^2 - \omega^2 - i\omega\gamma}$ where A is a positive constant, ω_0 the resonant frequency and γ the damping coefficient. For an electromagnetic wave of angular frequency $\omega \ll \omega_0$, which of the following is true? (Assume that $\frac{\gamma}{\omega_0} \ll 1$)
- (1) There is negligible absorption of the wave
 (2) The wave propagation is highly dispersive
 (3) There is strong absorption of the electromagnetic wave
 (4) The group velocity and the phase velocity will have opposite sign
56. A hydrogen atom is subjected to the perturbation $V_{pert}(r) = \epsilon \cos 2r/a_0$ where a_0 is the Bohr radius. The change in the ground state energy to first order in ϵ is
- (1) $\epsilon/4$ (2) $\epsilon/2$ (3) $-\epsilon/2$ (4) $-\epsilon/4$
57. A positron is suddenly absorbed by the nucleus of a tritium (${}^3_1\text{H}$) atom to turn the latter into a He^+ ion. If the electron in the tritium atom was initially in the ground state, the probability that the resulting He^+ ion will be in its ground state is
- (1) 1 (2) $\frac{8}{9}$ (3) $\frac{128}{243}$ (4) $\frac{512}{729}$

58. The product of the uncertainties $(\Delta L_x)(\Delta L_y)$ for a particle in the state $a|1,1\rangle + b|1,-1\rangle$ (where $|l,m\rangle$ denotes an eigenstate of L^2 and L_z) will be a minimum for

- (1) $a = \pm ib$ (2) $a = 0$ and $b = 1$
 (3) $a = \frac{\sqrt{3}}{2}$ and $b = \frac{1}{2}$ (4) $a = \pm b$

59. The ground state energy of a particle in the potential $V(x) = g|x|$, estimated using the trial wavefunction

$$\psi(x) = \begin{cases} \sqrt{\frac{c}{a^5}}(a^2 - x^2), & x < |a| \\ 0, & x \geq |a| \end{cases} \quad (\text{where } g \text{ and } c \text{ are constants})$$

- (1) $\frac{15}{16} \left(\frac{\hbar^2 g^2}{m} \right)^{1/3}$ (2) $\frac{5}{6} \left(\frac{\hbar^2 g^2}{m} \right)^{1/3}$ (3) $\frac{3}{4} \left(\frac{\hbar^2 g^2}{m} \right)^{1/3}$ (4) $\frac{7}{8} \left(\frac{\hbar^2 g^2}{m} \right)^{1/3}$

60. An ensemble of non-interacting spin- $\frac{1}{2}$ particles is in contact with a heat bath at temperature T and is subjected to an external magnetic field. Each particle can be in one of the two quantum states of energies $\pm \epsilon_0$. If the mean energy per particle is $-\epsilon_0/2$, then the free energy per particle is

- (1) $-2\epsilon_0 \frac{\ln(4/\sqrt{3})}{\ln 3}$ (2) $-\epsilon_0 \ln(3/2)$ (3) $-2\epsilon_0 \ln 2$ (4) $-\epsilon_0 \frac{\ln 2}{\ln 3}$

61. Which of the following graphs shows the qualitative dependence of the free energy $f(h, T)$ of a ferromagnet in an external magnetic field h , and at a fixed temperature $T < T_c$, where T_c is the critical temperature?

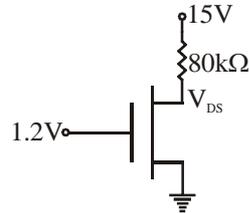


62. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than once is

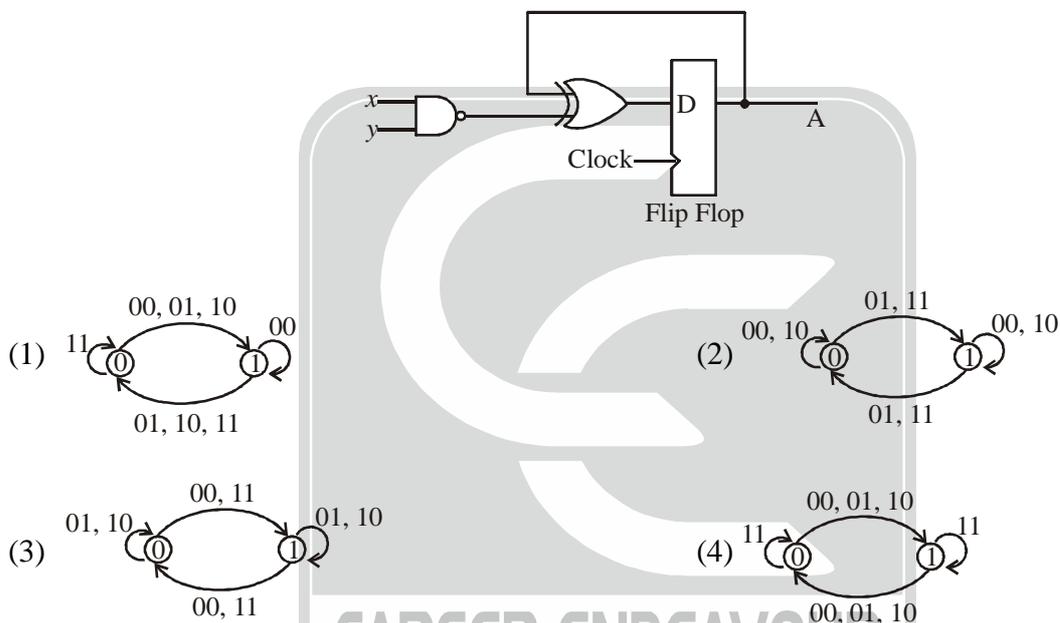
- (1) $12/27$ (2) $27/64$ (3) $3/8$ (4) $9/16$

63. Consider an n -MOSFET with the following parameters: current drive strength $K = 60 \mu A/V^2$, breakdown voltage $BV_{DS} = 10V$, ratio of effective gate width to the channel length $\frac{W}{L} = 5$ and threshold voltage $V_{th} = 0.5V$. In the circuit given below, this n -MOSFET is operating in the

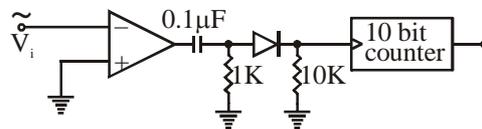
- (1) ohmic region
- (2) cut-off region
- (3) saturation region
- (4) breakdown region



64. The state diagram corresponding to the following circuit is



65. A sinusoidal signal of peak to peak amplitude 1V and unknown time period is input to the following circuit for 5 seconds duration. If the counter measures a value $(3E8)_H$ in hexadecimal then the time period of the input signal is



- (1) 2.5 ms
- (2) 4 ms
- (3) 10 ms
- (4) 5 ms

66. The first order diffraction peak of a crystalline solid occurs at a scattering angle of 30° when the diffraction pattern is recorded using an x -ray beam of wavelength 0.15nm. If the error in measurements of the wavelength and the angle are 0.01nm and 1° respectively, then the error in calculating the inter-planar spacing will approximately be

- (1) 1.1×10^{-2} nm
- (2) 1.3×10^{-4} nm
- (3) 2.5×10^{-2} nm
- (4) 2.0×10^{-3} nm



67. The dispersion relation of electrons in a 3-dimensional lattice in the tight binding approximation is given by,

$$\varepsilon_k = \alpha \cos k_x a + \beta \cos k_y a + \gamma \cos k_z a \text{ where } a \text{ is the lattice constant and } \alpha, \beta, \gamma \text{ are constants with}$$

dimension of energy. The effective mass tensor at the corner of the first Brillouin zone $\left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right)$

is

$$(1) \frac{\hbar^2}{a^2} \begin{pmatrix} -\frac{1}{\alpha} & 0 & 0 \\ 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$$

$$(2) \frac{\hbar^2}{a^2} \begin{pmatrix} -\frac{1}{\alpha} & 0 & 0 \\ 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & -\frac{1}{\gamma} \end{pmatrix}$$

$$(3) \frac{\hbar^2}{a^2} \begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$$

$$(4) \frac{\hbar^2}{a^2} \begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & -\frac{1}{\gamma} \end{pmatrix}$$

68. A thin metal film of dimension $2 \text{ mm} \times 2 \text{ mm}$ contains 4×10^{12} electrons. The magnitude of the Fermi wavevector of the system, in the free electron approximation, is

$$(1) 2\sqrt{\pi} \times 10^7 \text{ cm}^{-1}$$

$$(2) \sqrt{2\pi} \times 10^7 \text{ cm}^{-1}$$

$$(3) \sqrt{\pi} \times 10^7 \text{ cm}^{-1}$$

$$(4) 2\pi \times 10^7 \text{ cm}^{-1}$$

69. For an electron moving through a one-dimensional periodic lattice of periodicity a , which of the following corresponds to an energy eigenfunction consistent with Bloch's theorem?

$$(1) \psi(x) = A \exp\left(i\left[\frac{\pi x}{a} + \cos\left(\frac{\pi x}{2a}\right)\right]\right) \quad (2) \psi(x) = A \exp\left(i\left[\frac{\pi x}{a} + \cos\left(\frac{2\pi x}{a}\right)\right]\right)$$

$$(3) \psi(x) = A \exp\left(i\left[\frac{2\pi x}{a} + i \cos\left(\frac{2\pi x}{a}\right)\right]\right) \quad (4) \psi(x) = A \exp\left(i\left[\frac{\pi x}{2a} + i\left|\frac{\pi x}{2a}\right|\right]\right)$$

70. The LS configurations of the ground state of ^{12}Mg , ^{13}Al , ^{17}Cl and ^{18}Ar are, respectively.

$$(1) {}^3S_1, {}^2P_{1/2}, {}^2P_{1/2} \text{ and } {}^1S_0$$

$$(2) {}^3S_1, {}^2P_{3/2}, {}^2P_{3/2} \text{ and } {}^3S_1$$

$$(3) {}^1S_0, {}^2P_{1/2}, {}^2P_{3/2} \text{ and } {}^1S_0$$

$$(4) {}^1S_0, {}^2P_{3/2}, {}^2P_{1/2} \text{ and } {}^3S_1$$

71. For a two levels system, the population of atoms in the upper and lower levels are 3×10^{18} and 0.7×10^{18} , respectively. If the coefficient of stimulated emission is $3.0 \times 10^5 \text{ m}^3/\text{W}\cdot\text{s}^3$ and the energy density is $9.0 \text{ J/m}^3\cdot\text{Hz}$, the rate of stimulated emission will be

$$(1) 6.3 \times 10^{16} \text{ s}^{-1}$$

$$(2) 4.1 \times 10^{16} \text{ s}^{-1}$$

$$(3) 2.7 \times 10^{16} \text{ s}^{-1}$$

$$(4) 1.8 \times 10^{16} \text{ s}^{-1}$$

72. The first ionization potential of K is 4.34 eV, the electron affinity of Cl is 3.82 eV and the equilibrium separation of KCl is 0.3nm. The energy required to dissociate a KCl molecule into a K and a Cl atom is

- (1) 8.62 eV (2) 8.16 eV (3) 4.28 eV (4) 4.14 eV

73. Consider the following processes involving free particles

- (i) $\bar{n} \rightarrow \bar{p} + e^+ + \bar{\nu}_e$ (ii) $\bar{p} + n \rightarrow \pi^-$
 (iii) $p + n \rightarrow \pi^+ + \pi^0 + \pi^0$ (iv) $p + \bar{\nu}_e \rightarrow n + e^+$

Which of the following statements is true?

- (1) Process (i) obeys all conservation laws
 (2) Process (ii) conserves baryon number, but violates energy-momentum conservation
 (3) Process (iii) is not allowed by strong interactions, but is allowed by weak interactions
 (4) Process (iv) conserves baryon number, but violates lepton number conservation

74. The electric quadrupole moment of an odd proton nucleus is $\frac{(2j-1)}{2(j+1)} \langle r^2 \rangle$, where j is the total angular momentum. Given that $R_0 = 1.2$ fm, what is the value, in barn, of the quadrupole moment of the ^{27}Al nucleus in the shell model?

- (1) 0.043 (2) 0.023 (3) 0.915 (4) 0

75. Of the nuclei of mass number $A = 125$, the binding energy calculated from the liquid drop model (given that the coefficients for the Coulomb and the asymmetry energy are $a_c = 0.7$ MeV and $a_{sym} = 22.5$ MeV respectively) is a maximum for

- (1) $^{125}_{54}\text{Xe}$ (2) $^{125}_{53}\text{I}$ (3) $^{125}_{52}\text{Te}$ (4) $^{125}_{51}\text{Sb}$

