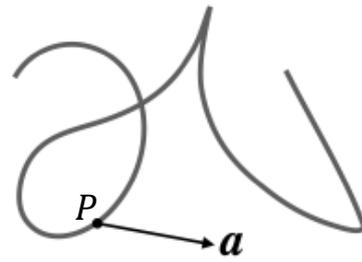


## GS 2026 Entrance exam, Physics, PhD: Section A

A1

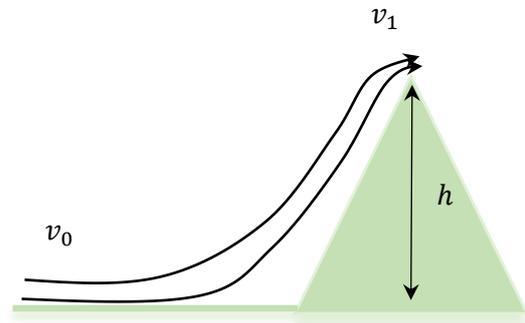
- (1) Is it possible for the trajectory of a particle in a two-dimensional plane with a continuous velocity and acceleration to have the shape shown in the figure?
- (2) Is it possible for its acceleration vector at  $P$  on the trajectory to point in the direction,  $\mathbf{a}$ , as shown?



- (a) (1) Yes (2) No
- (b) (1) Yes (2) Yes
- (c) (1) No (2) No
- (d) (1) No (2) Yes

A2

A steady, incompressible air stream of density  $\rho = 1.2 \text{ kg/m}^3$  blows horizontally at  $v_0 = 15 \text{ m/s}$  towards a hill of height  $h = 50 \text{ m}$ . Because the streamlines constrict over the hilltop, the air speed there increases to  $v_1 = 19.5 \text{ m/s}$ . The difference between the pressure at the base and the hilltop is closest to:



- (a) 681 N/m<sup>2</sup>
- (b) 495 N/m<sup>2</sup>
- (c) 588 N/m<sup>2</sup>
- (d) 93 N/m<sup>2</sup>

A3

A student performs an experiment to measure the acceleration due to gravity,  $g$ , using a simple pendulum. The student measures  $L = 1.00 \pm 0.01$  m and the time for 50 oscillations  $t_{50} = 100.0 \pm 1.0$  s. Based on this experiment, what is the calculated value of  $g$  and its uncertainty, assuming that the errors on the length and time are statistically independent?

(a)  $9.87 \pm 0.22 \text{ m/s}^2$

(b)  $9.87 \pm 0.11 \text{ m/s}^2$

(c)  $9.87 \pm 0.44 \text{ m/s}^2$

(d)  $9.87 \pm 0.05 \text{ m/s}^2$

A4 Consider a  $2 \times 2$  matrix defined as:

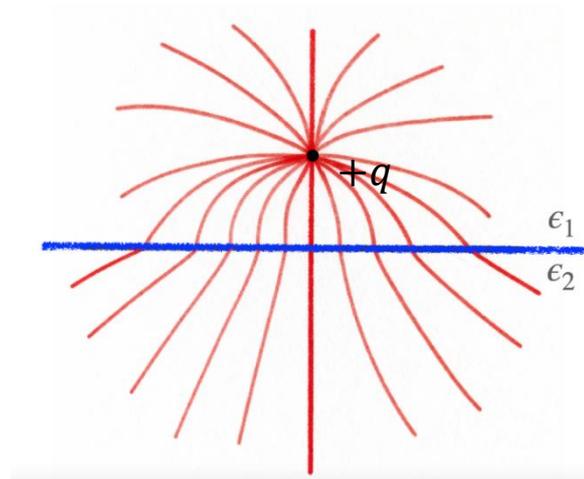
$$A = [a_0 + i \vec{\sigma} \cdot \vec{a}][a_0 - i \vec{\sigma} \cdot \vec{a}]^{-1}$$

where  $a_0$  is a non-zero real number,  $\vec{a}$  is a three-dimensional non-zero real vector, and  $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ , where  $\sigma_x, \sigma_y, \sigma_z$  are the Pauli spin matrices. The matrix  $A$ :

- (a) Is Unitary but not Hermitian
- (b) Is Hermitian but not Unitary
- (c) Is Unitary and Hermitian
- (d) Is neither Unitary nor Hermitian

A5

The figure on the right shows a positive point charge ( $+q$ ) placed above an interface (horizontal line in the figure) between two dielectrics with permittivities,  $\epsilon_1$  and  $\epsilon_2$ . The curves shown are field lines of the electric displacement,  $\vec{D}$ . Which of the following statements is true?



- (a)  $\epsilon_2 > \epsilon_1$  and the sign of the net surface charge at the interface is negative
- (b)  $\epsilon_1 > \epsilon_2$  and the sign of the net surface charge at the interface is positive
- (c)  $\epsilon_2 > \epsilon_1$  and the sign of the net surface charge at the interface is positive
- (d)  $\epsilon_1 > \epsilon_2$  and the sign of the net surface charge at the interface is negative

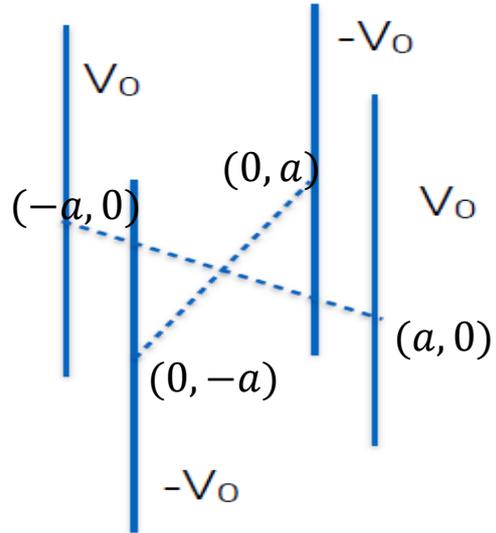
A6

A thin film of oil (refractive index,  $n_{oil} = 1.40$ ) floats on a puddle of water (refractive index,  $n_{water} = 1.33$ ) on a road. The film is illuminated at normal incidence from air by white light. In the light reflected vertically, green colour (with wavelength in air  $\lambda_0 = 560$  nm) is enhanced. The smallest possible thickness of the oil film is approximately:

- (a) 100 nm
- (b) 2000 nm
- (c) 200 nm
- (d) 360 nm

A7

Four infinitely long electrodes of negligible thickness are placed parallel to the  $z$  axis and pass through the points at the edges of a square in the  $(x, y)$  plane at  $(\pm a, 0)$  and  $(0, \pm a)$ . The potential at  $(0, 0)$  is 0. If the electrodes at  $(\pm a, 0)$  are maintained at a potential  $V_0$ , while the electrodes at  $(0, \pm a)$  are maintained at a potential  $-V_0$ , the electric field in the vicinity of the electrodes is given by:



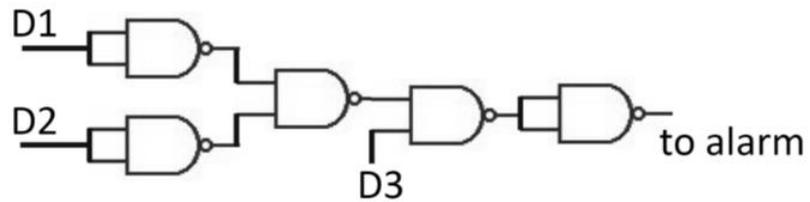
(a) 
$$\vec{E} = -\frac{2V_0}{a^2}(x\hat{x} - y\hat{y})$$

(b) 
$$\vec{E} = -\frac{V_0}{a}(\hat{x} - \hat{y})$$

(c) 
$$\vec{E} = \frac{2\pi V_0}{a^2}\left(\sin\left(\frac{2\pi x}{a}\right)\hat{x} - \sin\left(\frac{2\pi y}{a}\right)\hat{y}\right)$$

(d) 
$$\vec{E} = -\frac{V_0}{x^2 + y^2}(\hat{x} - \hat{y})$$

A8 Three nearby laboratory rooms **1, 2, 3** have one smoke detector each labelled **D1, D2, D3** respectively, for fire safety. In case smoke is detected, the detector output goes to a logic state *True*. However, to prevent false alarm, these outputs are connected to the logic circuit below whose output activates a single alarm.



In which of the following cases will the alarm definitely ring:

- (a) Smoke is detected in rooms 2 and 3
- (b) Smoke is detected in rooms 1 and 2
- (c) Smoke is detected in rooms 1 or 2
- (d) Smoke is detected in any two of three rooms

A9

Find:

$$\lim_{x \rightarrow +\infty} x \log \left( \frac{x+1}{x-1} \right)$$

(a)

2

(b)

1

(c)

0

(d)

The limit does not exist

A10

Scientists are conducting an underground experiment where two interactions,  $A$  and  $B$  can occur. In general, interaction  $A$  is twice as likely to occur compared to interaction  $B$ . Interaction  $A$  would give a signal in the detector with probability  $1/8$ , while interaction  $B$  would give a signal with probability  $1/2$ . If the detector registers a signal, what is the probability that the signal was due to interaction  $A$ ?

- (a)  $\frac{1}{3}$
- (b)  $\frac{1}{5}$
- (c)  $\frac{1}{9}$
- (d)  $\frac{2}{3}$

A11

The equation of state of a gas is given by:

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

where  $v$  is the volume per mole and  $R, a, b$  are constants. The internal energy per mole of the gas is given by:

$$U(T, v) = \frac{3}{2}RT - \frac{a}{v}$$

Which of the following gives the adiabatic equation of state at fixed particle number for the gas?

(a)  $(v - b)^2 T^3 = \text{constant}$

(b)  $(v - b)^2 T^{-3} = \text{constant}$

(c)  $(v - b)^2 T^{\frac{3}{2}} = \text{constant}$

(d)  $(v - b)^2 T^{-\frac{3}{2}} = \text{constant}$

A12

Electrons in a metal are accelerated under a constant electric field  $\vec{E}$  and experience a drag  $-\eta\vec{v}$  due to the surrounding medium. If the density of electrons is  $n$  and the current is constant, what is the conductivity  $\sigma$  of the metal?

(a)  $\sigma = \frac{ne^2}{\eta}$

(b)  $\sigma = \frac{ne^2E}{\eta}$

(c)  $\sigma = \frac{ne^2}{m\eta}$

(d)  $\sigma = \frac{neE}{m\eta}$

A13 *(Note: Either (a) or (b) can be correct, depending on the applications; both (a) or (b) will be given full marks.)*

A thermistor measures the temperature by measuring the change of electric resistivity of a material. Which of the following types of materials are most suited for making thermistors to work in the temperature range  $-100$  degree C to  $300$  degree C?

(a) Semiconductors

(b) Metals

(c) Superconductors

(d) Gas of atoms

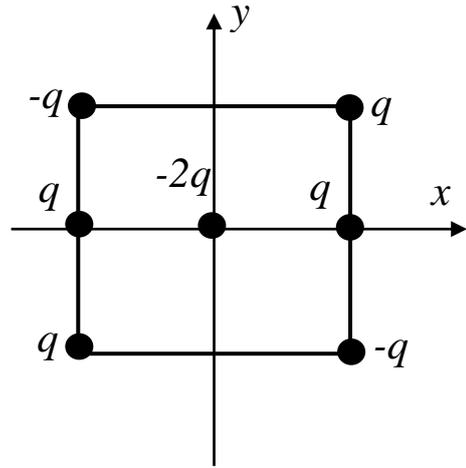
A14

An electron beam is accelerated over a potential  $V$  and strikes a crystal with lattice constant  $5\text{\AA}$ . What is the minimum  $V$  needed to resolve the crystal?

- (a) 5 V
- (b) 30 V
- (c) 300 V
- (d) 0.5 V

A15

The figure on the right, shows point charges arranged in the  $x, y$  plane on the centre, vertices, and some edges of a square with vertices at  $(\pm 1, \pm 1, 0)$  in standard units. Given the charge distribution in the figure, the electric field far away from the charges ( $r = \sqrt{x^2 + y^2 + z^2}$  denotes the distance from the origin) falls off as:

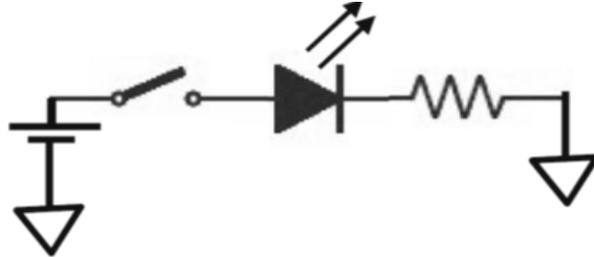


- (a)  $\frac{1}{r^4}$
- (b)  $\frac{1}{r^2}$
- (c)  $\frac{1}{r^3}$
- (d)  $\frac{1}{r^5}$

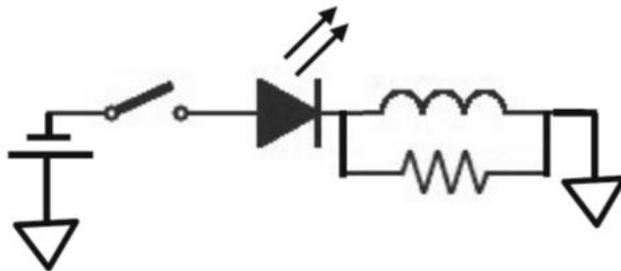
A16

Which of these circuits made with resistor, capacitor and inductor will be the best to get light out of the LED using a 9V battery, after the switch is closed?

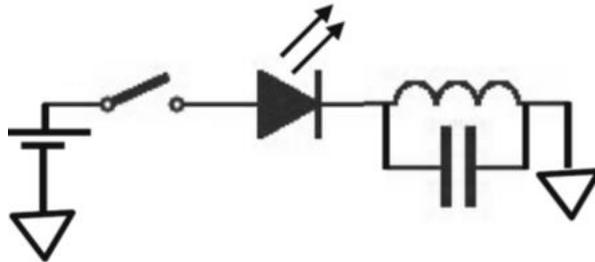
(a)



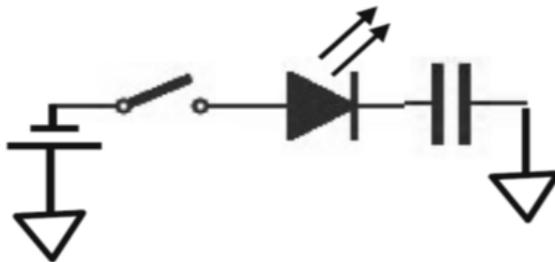
(b)



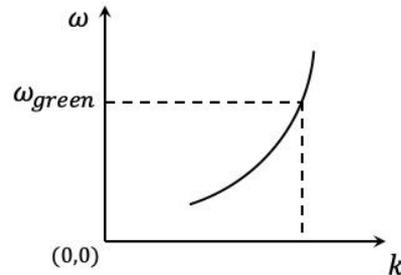
(c)



(d)



A17 For a medium, the dispersion relation for the propagation of light is shown below on a linear scale:



Here,  $\omega_{green}$  is the frequency of the green light. Which of the following statements is correct?

- (a) The phase velocity for the green light is smaller than the group velocity at that frequency
- (b) The refractive index of the medium for the red light is smaller than that for the blue light
- (c) The phase velocity of the green light is larger than the group velocity at that frequency
- (d) The speed of light in this medium for green light is same as that in vacuum

A18

Hydrogen atom has a decay rate from  $n = 2$  state to  $n = 1$  state as about  $6 \times 10^8$  decays per second. If the energy level expression for the hydrogen atom is

$$E_n = \frac{13.6}{n^2} \text{ eV}$$

the uncertainty in the wavelength of this emission would be closest to

- (a) 30 fm
- (b) 0.1 fm
- (c) 122 nm
- (d) 19.4 nm

A19

A quantum particle of mass  $m$  in three-dimensions is governed by the Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2 + z^2) + \lambda(x^6 + y^6)$$

with  $\lambda > 0$ . If  $\psi_3(\vec{r})$  is the third excited state of the system, which of the following is correct?

- (a)  $|\psi_3(\vec{r})|^2 = |\psi_3(-\vec{r})|^2$
- (b)  $\psi_3(\vec{r})$  is an eigenstate of  $L^2$
- (c)  $\psi_3(\vec{r})$  is an eigenstate of  $L_z$
- (d)  $\psi_3(\vec{r})$  is an eigenstate of  $p_z$

A20

Consider the series:

$$S = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

Which of the following statements is correct?

- (a)  $S$  is convergent and is greater than 2
- (b)  $S$  is convergent and is less than 2
- (c)  $S$  is convergent and is equal to 2
- (d)  $S$  is not convergent

- A21 A relativistic particle of mass  $m$  moving under the central force of gravity with angular momentum  $L$  around a massive body of mass  $M$  experiences the following potential:

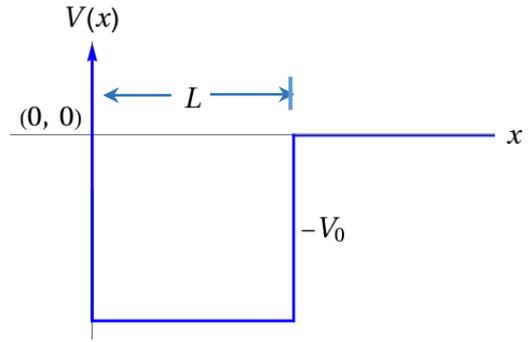
$$V(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{m c^2 r^3}$$

where the last term is the relativistic correction to the Newtonian formula. For sufficiently large  $L$ , the particle has:

- (a) Two circular orbits. The smaller one is unstable and the larger is stable
- (b) Two circular orbits. The smaller one is stable and the larger is unstable
- (c) Three circular orbits. The middle one is unstable and the others stable
- (d) Three circular orbits. The middle one is stable and the others unstable

A22

Consider a quantum particle in a one-dimensional potential of the shape shown, with the well of width  $L$ , and  $V(x < 0) \rightarrow \infty$ . The potential in the well is  $-V_0$  and the potential for  $x \in (L, \infty)$  is 0. Which plot describes the change in the ground state energy ( $E_g$ ) as  $L$  is changed keeping  $V_0$  constant?



- (a)
- (b)
- (c)
- (d)

A23

A non-interacting, classical gas is made up of  $N$  Nitrogen molecules. The specific heat at constant volume is  $C_v$ .  $C_v/(Nk_B T)$  is given by:

(Ignore rotations about the axis of symmetry.)

- (a)  $3/2$  for some very low  $T$ ,  $5/2$  for temperatures around the room temperature,  $7/2$  for some high temperatures
- (b)  $7/2$  for some very low  $T$ ,  $5/2$  for temperatures around the room temperature,  $3/2$  for some high temperatures
- (c)  $3/2$  for some very low  $T$ ,  $2$  for temperatures around the room temperature,  $5/2$  for some high temperatures
- (d)  $3/2$  for some very low  $T$ ,  $2$  for temperatures around the room temperature,  $3$  for some high temperatures

A24

The longitudinal disturbance generated by an earthquake, travels through the earth's crust and reaches 1000 km in 3 mins from the epicentre of the earthquake. Assuming the density of the Earth's crust is  $2700 \text{ kg/m}^3$ , the Bulk's modulus of the crust is closest to:

(Ignore the shear modulus and local variations in the density and the Bulk modulus.)

(a)  $8.3 \times 10^{10} \text{ N m}^{-2}$

(b)  $2.1 \times 10^{11} \text{ N m}^{-2}$

(c)  $8.3 \times 10^8 \text{ N m}^{-2}$

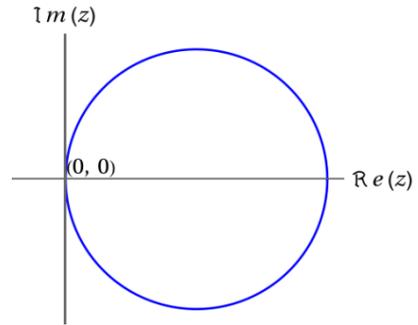
(d)  $2.1 \times 10^9 \text{ N m}^{-2}$

A25

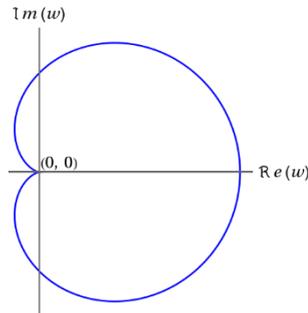
An analytic transformation:

$$w = z^2$$

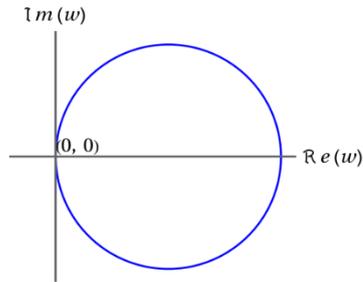
is applied on the complex plane. Consider the circle  $C$  in  $z$  as shown on the right. Which of the following represents the image of  $C$  in the  $w$  plane?



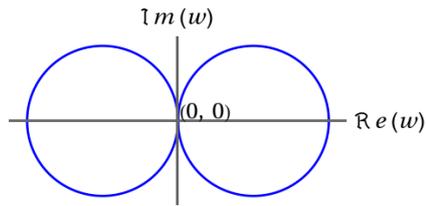
(a)



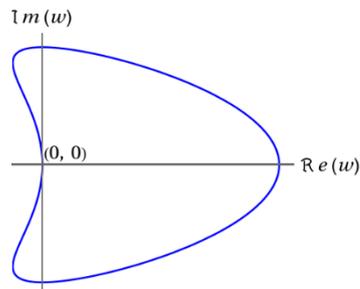
(b)



(c)



(d)



## GS 2026, Physics, PhD: Section C

C1

Consider a quantum particle moving in one spatial dimension with the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

Let  $|\psi(t)\rangle$  be the state of the system where  $|\psi(0)\rangle$  is a normalized eigenstate of this Hamiltonian. What is the value of  $\langle\psi(t)|\hat{x}\hat{p}|\psi(t)\rangle$ ? (Hint: Consider  $\frac{d\langle\psi(t)|\hat{x}^2|\psi(t)\rangle}{dt}$ .)

- (a)  $\frac{i\hbar}{2}$
- (b)  $-i\hbar$
- (c)  $\hbar$
- (d)  $0$

C2

In the standard  $|S^2, S_z\rangle$  eigenbasis of a spin 1 particle, the matrix corresponding to the operator  $\hat{S}_x^2 - \hat{S}_y^2$  is given by:

(a) 
$$\hbar^2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

(b) 
$$\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c) 
$$\hbar^2 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(d) 
$$\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

C3

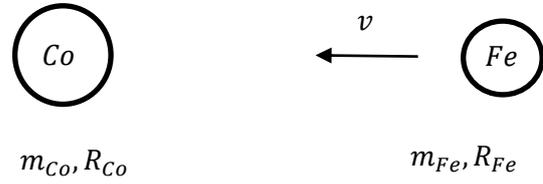
The Hamiltonian of a quantum system in the basis states,  $|1\rangle$ ,  $|2\rangle$ , and  $|3\rangle$  is given by the matrix:

$$\omega_0 \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

If the system is in the state  $|\psi(0)\rangle = (|1\rangle - |3\rangle)/\sqrt{2}$  at  $t = 0$ , the probability of the system to be in state  $|2\rangle$  at time  $t$  is given by:

- (a) 0
- (b)  $\sin^2(2\omega_0 t)$
- (c)  $\sin^2(\omega_0 t)$
- (d)  $\sin^2\left(\frac{\omega_0 t}{2}\right)$

C4 An *Fe* ( $A=56, Z=26$ ) nucleus starts from infinity with an initial speed  $v$  and moves head on towards a *Co* ( $A=59, Z=27$ ) nucleus which is initially stationary. The nuclei are uniformly charged spheres of radii  $R = 1.2 A^{1/3} \text{fm}$ , and a nuclear reaction can take place when they touch each other. What is the smallest  $v$  for which this reaction can occur? Neglect the mass difference between neutrons and protons and nuclear forces, and assume that the two nuclei remain spherical throughout the motion.



- (a)  $2.70 \times 10^7 \text{m/s}$
- (b)  $1.91 \times 10^7 \text{m/s}$
- (c)  $1.37 \times 10^7 \text{m/s}$
- (d)  $0.95 \times 10^7 \text{m/s}$

C5

A classical particle of mass  $m$  moving in the  $x - y$  plane is constrained to move along the curve  $x^4 + y^4 = 1$ . There are no other external forces acting on it. If  $(r, \theta)$  are the polar co-ordinates, which of the following is true?

(a) 
$$\ddot{\theta} + 2 \frac{\dot{r}\dot{\theta}}{r} \propto r^2 \sin 4\theta$$

(b) 
$$\ddot{\theta} \propto r^2 \sin 4\theta$$

(c) 
$$\ddot{\theta} \propto r^4 \sin^2 2\theta$$

(d) 
$$\ddot{\theta} + 2 \frac{\dot{r}\dot{\theta}}{r} \propto r^4 \sin^2 2\theta$$

C6

A particle of rest mass  $m$  moving at a relativistic speed  $v$  collides with an identical particle at rest and merges with it to form a composite of rest mass  $M = \frac{4}{\sqrt{3}}m$ . What is the ratio of the speed of  $M$  to the initial speed  $v$ ?

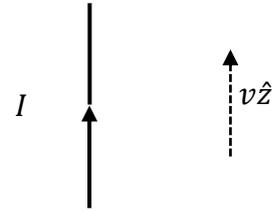
(a)  $\frac{5}{8}$

(b)  $\frac{1}{2}$

(c)  $\frac{3}{5}$

(d)  $\frac{2}{3}$

C7 For a static observer, an infinite wire of negligible thickness with no net charge density placed along  $z$  direction carries a current  $I$ . Another observer is moving along the current with a relativistic speed  $v$ . The magnitude of the linear charge density on the wire seen by the moving observer is:



(a) 
$$\frac{Iv}{c\sqrt{c^2 - v^2}}$$

(b) 
$$\frac{I}{\sqrt{c^2 - v^2}}$$

(c) 
$$\frac{I\sqrt{c^2 - v^2}}{vc}$$

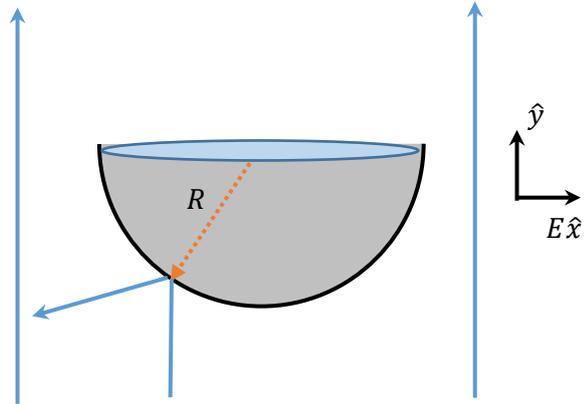
(d) 
$$0$$

C8

A plane electromagnetic wave with an electric field

$$\vec{E}(t, \vec{r}) = E \hat{x} \cos(ky - \omega t)$$

is incident on a hemispherical mirror of radius  $R$  with a perfectly reflecting outer surface, placed as shown.



What is the electromagnetic force acting on the mirror averaged over one time-period of the wave?

(a)

$$\frac{\pi \epsilon_0 E^2 R^2}{2}$$

(b)

$$\frac{2\pi \epsilon_0 E^2 R^2}{3}$$

(c)

$$\pi \epsilon_0 E^2 R^2$$

(d)

$$\frac{4\pi \epsilon_0 E^2 R^2}{3}$$

C9

The inner product of two complex functions  $f(x), g(x)$ , over the interval  $x \in [0,1]$  is defined as:

$$\langle f|g \rangle = \int_0^1 dx x(1-x)f^*(x)g(x)$$

The adjoint,  $\hat{O}^\dagger$ , of a linear operator  $\hat{O}$  is defined by the equation:

$$\langle f|\hat{O}g \rangle = \langle \hat{O}^\dagger f|g \rangle$$

What is the adjoint of the operator  $\hat{O} = \frac{d}{dx}$ ? Assume that both  $f(x)$  and  $g(x)$  are differentiable.

(a)  $\frac{2x-1}{x(1-x)} - \frac{d}{dx}$

(b)  $-\frac{d}{dx}$

(c)  $\frac{1}{x(1-x)} \frac{d}{dx}$

(d)  $\frac{-1}{x(1-x)} \frac{d}{dx}$

C10

What is the value of the following integral:

$$\int_0^{\infty} dx \frac{\sin x}{x} \frac{1}{x^2 + a^2}$$

(Assume  $a > 0$ .)

(a)  $\frac{\pi}{2a^2} (1 - e^{-a})$

(b)  $\frac{\pi}{2a^2} (1 + e^{-a})$

(c)  $\frac{\pi \sinh a}{2a^2}$

(d)  $\frac{\pi \sin a}{2a^2}$

C11

Consider a single particle in a potential well with an energy spectrum  $E_n = n\epsilon$ , for  $n = 0, 1, 2, 3, \dots$ , with  $\epsilon > 0$ . The  $n^{\text{th}}$  energy level has a degeneracy  $g(n) = 2n + 1$ . This system is at temperature  $T$ . What is the value of  $\langle \hat{H} \rangle$ ?

( $k$  is the Boltzmann constant.)

(a) 
$$\epsilon \frac{3e^{\frac{\epsilon}{kT}} + 1}{\left(e^{\frac{2\epsilon}{kT}} - 1\right)}$$

(b) 
$$\epsilon \frac{1}{\left(e^{\frac{\epsilon}{kT}} - 1\right)}$$

(c) 
$$\epsilon \frac{3e^{\frac{\epsilon}{kT}} + 1}{\left(e^{\frac{\epsilon}{kT}} - 1\right)^2}$$

(d) 
$$\epsilon \frac{1}{\left(e^{\frac{\epsilon}{kT}} - 1\right)^2}$$

C12

A system of  $N$  classical non-interacting particles of mass  $m$  is confined to a cubic box of volume  $V$ . Inside the box is a region of volume  $V_0 (< V)$ , where a constant potential  $U$  is present.

If  $P$  is the pressure of the system,  $T$  is its temperature, then:

(a) 
$$\frac{P}{Nk_B T} = \frac{1}{V - V_0(1 - e^{-U/k_B T})}$$

(b) 
$$\frac{P}{Nk_B T} = \frac{1}{V - V_0 e^{-U/k_B T}}$$

(c) 
$$\frac{P}{Nk_B T} = \frac{1}{V_0 e^{-U/k_B T}}$$

(d) 
$$\frac{P}{Nk_B T} = \frac{1}{V - V_0}$$

C13

A Bosonic excitation has a density of states given by:

$$\rho(E) = \rho_0 \left[ 1 - \left( \frac{E}{E_0} - 1 \right)^2 \right]$$

for  $0 \leq E \leq 2E_0$ , and 0 otherwise. If the two-dimensional system is in a temperature range  $k_B T \gg E_0$ , the variation of the specific heat with temperature is given by:

(a)  $C_V \propto T^0$

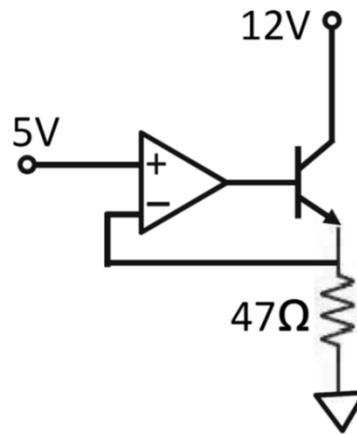
(b)  $C_V \propto T^3$

(c)  $C_V \propto T^2$

(d)  $C_V \propto T$

C14

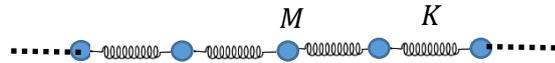
Op-Amps normally cannot output very high currents, which is typically limited to tens of mA. To overcome this, a transistor is used to boost the current output onto a load resistor. The circuit below uses a Si transistor with common-emitter mode current gain  $\beta = 120$ . What is approximately the current being drawn from the output of the Op-Amp here?



- (a) 0.88 mA
- (b) 0.76 mA
- (c) 1.23 mA
- (d) 2.11 mA

C15

Consider a one-dimensional lattice where each site has mass  $M$  and the lattice constant is  $a$ . Nearest neighbours are connected by springs of constant  $K$ . Which of the following best represents the dispersion of longitudinal oscillations in the lattice?



- (a)
- (b)
- (c)
- (d)