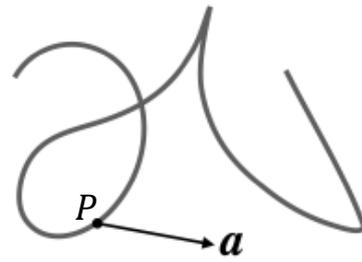


GS 2026 Entrance exam, Physics, Int-PhD: Section A

A1

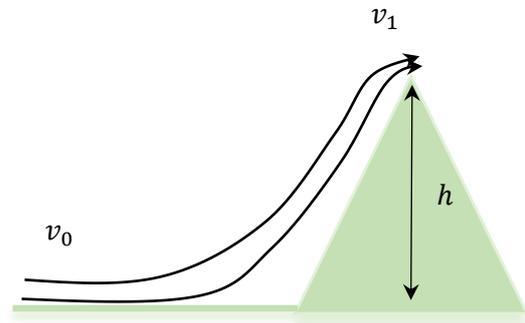
- (1) Is it possible for the trajectory of a particle in a two-dimensional plane with a continuous velocity and acceleration to have the shape shown in the figure?
- (2) Is it possible for its acceleration vector at P on the trajectory to point in the direction, \mathbf{a} , as shown?



- (a) (1) Yes (2) No
- (b) (1) Yes (2) Yes
- (c) (1) No (2) No
- (d) (1) No (2) Yes

A2

A steady, incompressible air stream of density $\rho = 1.2 \text{ kg/m}^3$ blows horizontally at $v_0 = 15 \text{ m/s}$ towards a hill of height $h = 50 \text{ m}$. Because the streamlines constrict over the hilltop, the air speed there increases to $v_1 = 19.5 \text{ m/s}$. The difference between the pressure at the base and the hilltop is closest to:



- (a) 681 N/m²
- (b) 495 N/m²
- (c) 588 N/m²
- (d) 93 N/m²

A3

A student performs an experiment to measure the acceleration due to gravity, g , using a simple pendulum. The student measures $L = 1.00 \pm 0.01$ m and the time for 50 oscillations $t_{50} = 100.0 \pm 1.0$ s. Based on this experiment, what is the calculated value of g and its uncertainty, assuming that the errors on the length and time are statistically independent?

(a) $9.87 \pm 0.22 \text{ m/s}^2$

(b) $9.87 \pm 0.11 \text{ m/s}^2$

(c) $9.87 \pm 0.44 \text{ m/s}^2$

(d) $9.87 \pm 0.05 \text{ m/s}^2$

A4 Consider a 2×2 matrix defined as:

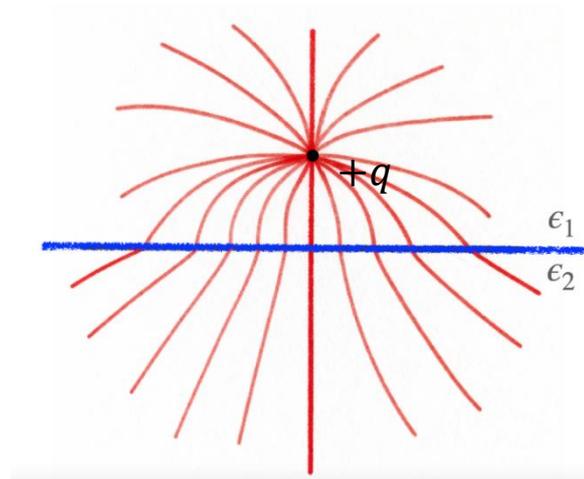
$$A = [a_0 + i \vec{\sigma} \cdot \vec{a}][a_0 - i \vec{\sigma} \cdot \vec{a}]^{-1}$$

where a_0 is a non-zero real number, \vec{a} is a three-dimensional non-zero real vector, and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli spin matrices. The matrix A :

- (a) Is Unitary but not Hermitian
- (b) Is Hermitian but not Unitary
- (c) Is Unitary and Hermitian
- (d) Is neither Unitary nor Hermitian

A5

The figure on the right shows a positive point charge ($+q$) placed above an interface (horizontal line in the figure) between two dielectrics with permittivities, ϵ_1 and ϵ_2 . The curves shown are field lines of the electric displacement, \vec{D} . Which of the following statements is true?



- (a) $\epsilon_2 > \epsilon_1$ and the sign of the net surface charge at the interface is negative
- (b) $\epsilon_1 > \epsilon_2$ and the sign of the net surface charge at the interface is positive
- (c) $\epsilon_2 > \epsilon_1$ and the sign of the net surface charge at the interface is positive
- (d) $\epsilon_1 > \epsilon_2$ and the sign of the net surface charge at the interface is negative

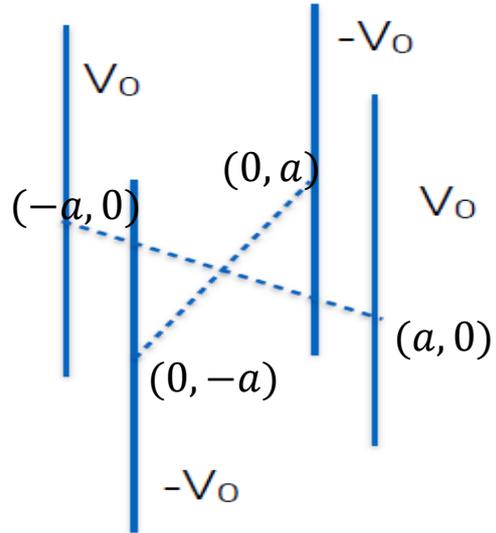
A6

A thin film of oil (refractive index, $n_{oil} = 1.40$) floats on a puddle of water (refractive index, $n_{water} = 1.33$) on a road. The film is illuminated at normal incidence from air by white light. In the light reflected vertically, green colour (with wavelength in air $\lambda_0 = 560$ nm) is enhanced. The smallest possible thickness of the oil film is approximately:

- (a) 100 nm
- (b) 2000 nm
- (c) 200 nm
- (d) 360 nm

A7

Four infinitely long electrodes of negligible thickness are placed parallel to the z axis and pass through the points at the edges of a square in the (x, y) plane at $(\pm a, 0)$ and $(0, \pm a)$. The potential at $(0, 0)$ is 0. If the electrodes at $(\pm a, 0)$ are maintained at a potential V_0 , while the electrodes at $(0, \pm a)$ are maintained at a potential $-V_0$, the electric field in the vicinity of the electrodes is given by:



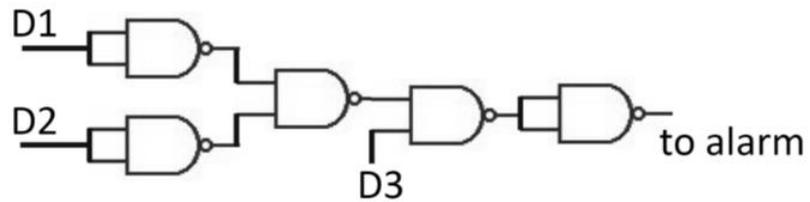
(a)
$$\vec{E} = -\frac{2V_0}{a^2}(x\hat{x} - y\hat{y})$$

(b)
$$\vec{E} = -\frac{V_0}{a}(\hat{x} - \hat{y})$$

(c)
$$\vec{E} = \frac{2\pi V_0}{a^2}\left(\sin\left(\frac{2\pi x}{a}\right)\hat{x} - \sin\left(\frac{2\pi y}{a}\right)\hat{y}\right)$$

(d)
$$\vec{E} = -\frac{V_0}{x^2 + y^2}(\hat{x} - \hat{y})$$

A8 Three nearby laboratory rooms **1, 2, 3** have one smoke detector each labelled **D1, D2, D3** respectively, for fire safety. In case smoke is detected, the detector output goes to a logic state *True*. However, to prevent false alarm, these outputs are connected to the logic circuit below whose output activates a single alarm.



In which of the following cases will the alarm definitely ring:

- (a) Smoke is detected in rooms 2 and 3
- (b) Smoke is detected in rooms 1 and 2
- (c) Smoke is detected in rooms 1 or 2
- (d) Smoke is detected in any two of three rooms

A9

Find:

$$\lim_{x \rightarrow +\infty} x \log \left(\frac{x+1}{x-1} \right)$$

(a)

2

(b)

1

(c)

0

(d)

The limit does not exist

A10

Scientists are conducting an underground experiment where two interactions, A and B can occur. In general, interaction A is twice as likely to occur compared to interaction B . Interaction A would give a signal in the detector with probability $1/8$, while interaction B would give a signal with probability $1/2$. If the detector registers a signal, what is the probability that the signal was due to interaction A ?

- (a) $\frac{1}{3}$
- (b) $\frac{1}{5}$
- (c) $\frac{1}{9}$
- (d) $\frac{2}{3}$

A11

The equation of state of a gas is given by:

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

where v is the volume per mole and R, a, b are constants. The internal energy per mole of the gas is given by:

$$U(T, v) = \frac{3}{2}RT - \frac{a}{v}$$

Which of the following gives the adiabatic equation of state at fixed particle number for the gas?

(a) $(v - b)^2 T^3 = \text{constant}$

(b) $(v - b)^2 T^{-3} = \text{constant}$

(c) $(v - b)^2 T^{\frac{3}{2}} = \text{constant}$

(d) $(v - b)^2 T^{-\frac{3}{2}} = \text{constant}$

A12

Electrons in a metal are accelerated under a constant electric field \vec{E} and experience a drag $-\eta\vec{v}$ due to the surrounding medium. If the density of electrons is n and the current is constant, what is the conductivity σ of the metal?

(a)
$$\sigma = \frac{ne^2}{\eta}$$

(b)
$$\sigma = \frac{ne^2E}{\eta}$$

(c)
$$\sigma = \frac{ne^2}{m\eta}$$

(d)
$$\sigma = \frac{neE}{m\eta}$$

A13 *(Note: Either (a) or (b) can be correct, depending on the applications; both (a) or (b) will be given full marks.)*

A thermistor measures the temperature by measuring the change of electric resistivity of a material. Which of the following types of materials are most suited for making thermistors to work in the temperature range -100 degree C to 300 degree C?

(a) Semiconductors

(b) Metals

(c) Superconductors

(d) Gas of atoms

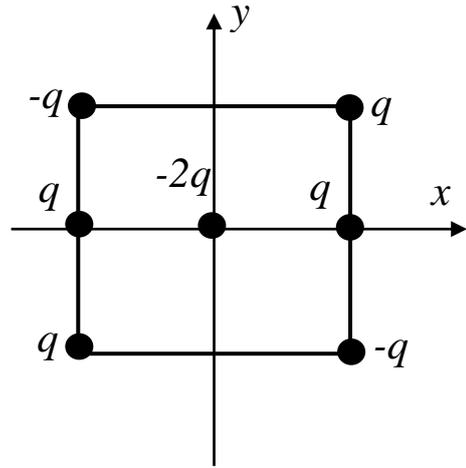
A14

An electron beam is accelerated over a potential V and strikes a crystal with lattice constant 5\AA . What is the minimum V needed to resolve the crystal?

- (a) 5 V
- (b) 30 V
- (c) 300 V
- (d) 0.5 V

A15

The figure on the right, shows point charges arranged in the x, y plane on the centre, vertices, and some edges of a square with vertices at $(\pm 1, \pm 1, 0)$ in standard units. Given the charge distribution in the figure, the electric field far away from the charges ($r = \sqrt{x^2 + y^2 + z^2}$ denotes the distance from the origin) falls off as:

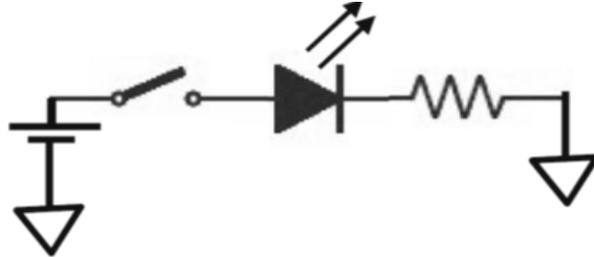


- (a) $\frac{1}{r^4}$
- (b) $\frac{1}{r^2}$
- (c) $\frac{1}{r^3}$
- (d) $\frac{1}{r^5}$

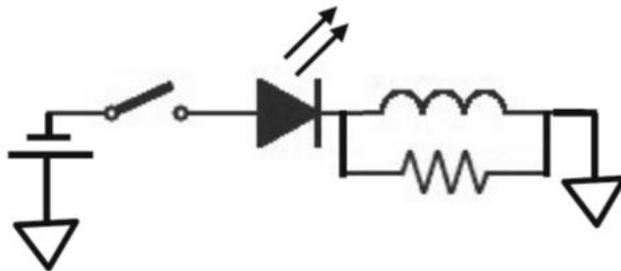
A16

Which of these circuits made with resistor, capacitor and inductor will be the best to get light out of the LED using a 9V battery, after the switch is closed?

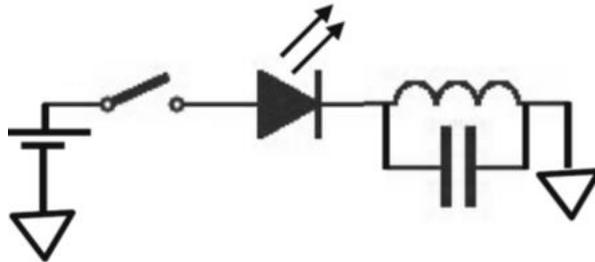
(a)



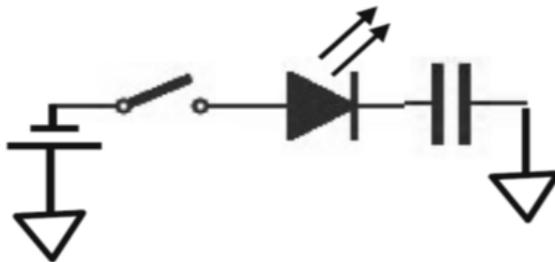
(b)



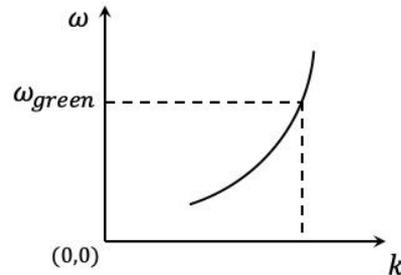
(c)



(d)



A17 For a medium, the dispersion relation for the propagation of light is shown below on a linear scale:



Here, ω_{green} is the frequency of the green light. Which of the following statements is correct?

- (a) The phase velocity for the green light is smaller than the group velocity at that frequency
- (b) The refractive index of the medium for the red light is smaller than that for the blue light
- (c) The phase velocity of the green light is larger than the group velocity at that frequency
- (d) The speed of light in this medium for green light is same as that in vacuum

A18

Hydrogen atom has a decay rate from $n = 2$ state to $n = 1$ state as about 6×10^8 decays per second. If the energy level expression for the hydrogen atom is

$$E_n = \frac{13.6}{n^2} \text{ eV}$$

the uncertainty in the wavelength of this emission would be closest to

- (a) 30 fm
- (b) 0.1 fm
- (c) 122 nm
- (d) 19.4 nm

A19

A quantum particle of mass m in three-dimensions is governed by the Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2 + z^2) + \lambda(x^6 + y^6)$$

with $\lambda > 0$. If $\psi_3(\vec{r})$ is the third excited state of the system, which of the following is correct?

- (a) $|\psi_3(\vec{r})|^2 = |\psi_3(-\vec{r})|^2$
- (b) $\psi_3(\vec{r})$ is an eigenstate of L^2
- (c) $\psi_3(\vec{r})$ is an eigenstate of L_z
- (d) $\psi_3(\vec{r})$ is an eigenstate of p_z

A20

Consider the series:

$$S = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

Which of the following statements is correct?

- (a) S is convergent and is greater than 2
- (b) S is convergent and is less than 2
- (c) S is convergent and is equal to 2
- (d) S is not convergent

- A21 A relativistic particle of mass m moving under the central force of gravity with angular momentum L around a massive body of mass M experiences the following potential:

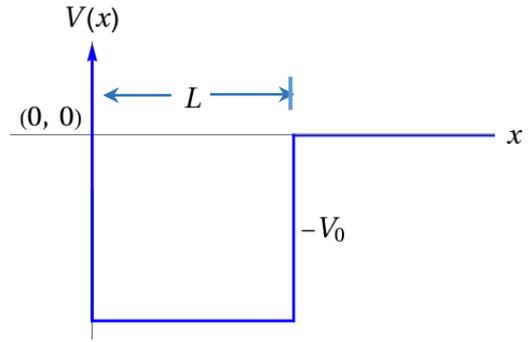
$$V(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{m c^2 r^3}$$

where the last term is the relativistic correction to the Newtonian formula. For sufficiently large L , the particle has:

- (a) Two circular orbits. The smaller one is unstable and the larger is stable
- (b) Two circular orbits. The smaller one is stable and the larger is unstable
- (c) Three circular orbits. The middle one is unstable and the others stable
- (d) Three circular orbits. The middle one is stable and the others unstable

A22

Consider a quantum particle in a one-dimensional potential of the shape shown, with the well of width L , and $V(x < 0) \rightarrow \infty$. The potential in the well is $-V_0$ and the potential for $x \in (L, \infty)$ is 0. Which plot describes the change in the ground state energy (E_g) as L is changed keeping V_0 constant?



- (a)
- (b)
- (c)
- (d)

A23

A non-interacting, classical gas is made up of N Nitrogen molecules. The specific heat at constant volume is C_v . $C_v/(Nk_B T)$ is given by:

(Ignore rotations about the axis of symmetry.)

- (a) $3/2$ for some very low T , $5/2$ for temperatures around the room temperature, $7/2$ for some high temperatures
- (b) $7/2$ for some very low T , $5/2$ for temperatures around the room temperature, $3/2$ for some high temperatures
- (c) $3/2$ for some very low T , 2 for temperatures around the room temperature, $5/2$ for some high temperatures
- (d) $3/2$ for some very low T , 2 for temperatures around the room temperature, 3 for some high temperatures

A24

The longitudinal disturbance generated by an earthquake, travels through the earth's crust and reaches 1000 km in 3 mins from the epicentre of the earthquake. Assuming the density of the Earth's crust is 2700 kg/m^3 , the Bulk's modulus of the crust is closest to:

(Ignore the shear modulus and local variations in the density and the Bulk modulus.)

(a) $8.3 \times 10^{10} \text{ N m}^{-2}$

(b) $2.1 \times 10^{11} \text{ N m}^{-2}$

(c) $8.3 \times 10^8 \text{ N m}^{-2}$

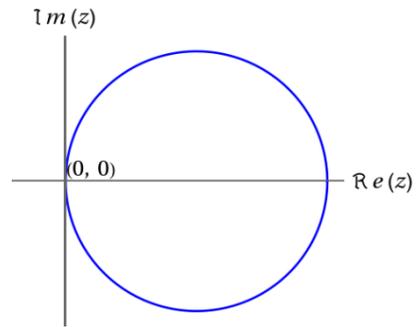
(d) $2.1 \times 10^9 \text{ N m}^{-2}$

A25

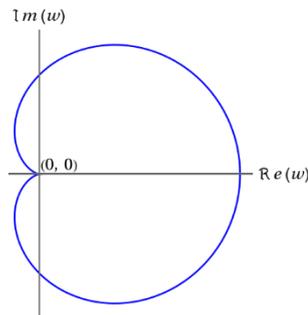
An analytic transformation:

$$w = z^2$$

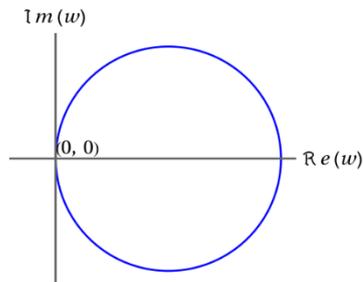
is applied on the complex plane. Consider the circle C in z as shown on the right. Which of the following represents the image of C in the w plane?



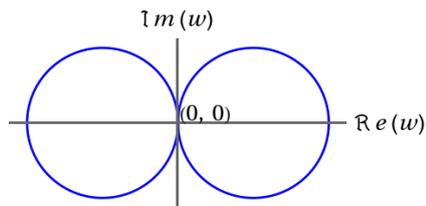
(a)



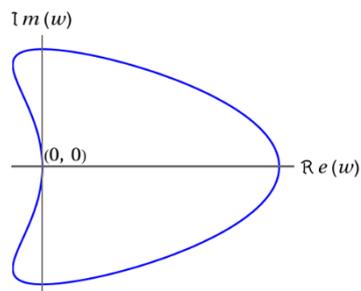
(b)



(c)



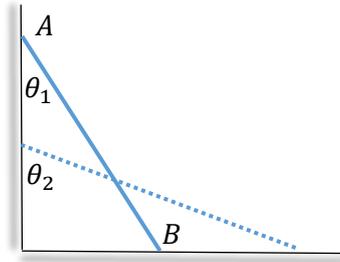
(d)



GS 2026, Physics, Int-PhD: Section B

B1

Consider a uniform rod with length L and mass m with one end A on the wall and the other end B on the ground. Initially it is at rest at an angle θ_1 with the wall and starts slipping. What is the speed of A when the ladder's angle with the wall is θ_2 ? Ignore friction.

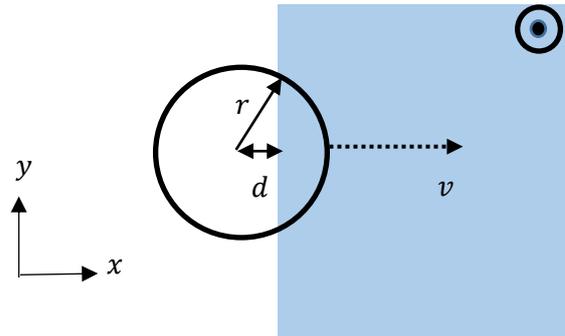


(FYI: The moment of inertia of a uniform rod about its centre is $I = \frac{1}{12} mL^2$.)

- (a) $\sin \theta_2 \sqrt{3gL(\cos \theta_1 - \cos \theta_2)}$
- (b) $\sqrt{3gL(\cos \theta_1 - \cos \theta_2)}$
- (c) $\sqrt{12gL(\cos \theta_1 - \cos \theta_2)}$
- (d) $\sin \theta_2 \sqrt{6gL(\cos \theta_1 - \cos \theta_2)}$

B2

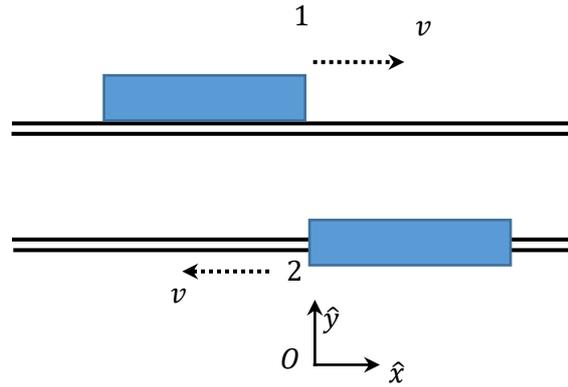
A metallic ring of radius r is pushed into a region of uniform magnetic field $B\hat{z}$ (shaded region) with a constant speed v as shown. The magnetic field is 0 outside. The electrical resistance of the loop is R . What is the external force one needs to apply on the loop to maintain its speed, when the distance of the region's edge to the centre is d (>0)?



- (a)
$$\frac{4r^2 B^2 v}{R} \left(1 - \frac{d^2}{r^2}\right) \hat{x}$$
- (b)
$$\frac{\pi r^2 B^2 v}{R} \left(1 + \frac{d^2}{r^2}\right) \hat{x}$$
- (c)
$$\frac{r^2 B^2 v}{R} \left(1 + \frac{d}{r}\right) \hat{x}$$
- (d)
$$\frac{\pi r^2 B^2 v}{R} \left(1 - \frac{d}{r}\right) \hat{x}$$

B3

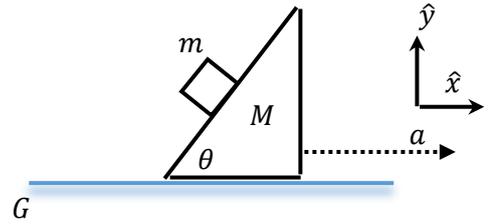
Two trains of proper length L each move on neighbouring tracks. An observer at rest with the tracks, O , notes the speed of train 1 to be $v\hat{x}$ and train 2 to be $-v\hat{x}$. O also notes that at $t = 0$ the front ends of the two trains have the same x position, $x = 0$. According to a train engineer sitting at the front of train 1, what is the time interval between the passing by of the front and the back of train 2?



- (a) $\frac{L}{2v}(1 - v^2/c^2)$
- (b) $\frac{L}{2v}\sqrt{1 - v^2/c^2}$
- (c) $\frac{L}{2v}\sqrt{\frac{1 - v^2/c^2}{1 + v^2/c^2}}$
- (d) $\frac{L}{2v}$

B4

A mass m is placed on a wedge of mass M and angle θ as shown, which in turn is placed on the horizontal ground. Initially, m and M are at rest with respect to the ground and are released from this position. What is the acceleration a of M ? Assume that all surfaces are frictionless.



(a)
$$a = \frac{mg \cos \theta \sin \theta}{M + m \sin^2 \theta}$$

(b)
$$a = \frac{mg \cos \theta}{M + m}$$

(c)
$$a = \frac{mg \sin \theta}{M + m}$$

(d)
$$a = 0$$

B5 Scientists have discovered a new type of particle, which can occupy a quantum state with 0, 1 or 2 particles. If a quantum state has energy ω , the average occupation of this state in a system at temperature T and chemical potential μ is given by

(a)
$$\frac{2 + e^{\frac{(\omega-\mu)}{T}}}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}}$$

(b)
$$\frac{2}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}}$$

(c)
$$\frac{1 + 2e^{\frac{(\omega-\mu)}{T}}}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}}$$

(d)
$$\frac{1}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}}$$

B6 A cycle for an ideal gas consists of the following three reversible processes:

i. Isothermal expansion at T_H from V_1 to V_2 .

ii. Isochoric (constant volume) cooling from T_H to T_C .

iii. Adiabatic compression from T_C and V_2 back to the initial state (T_H, V_1).

If C_V is the constant volume heat capacity of the gas, the efficiency η of this cycle is:

(a)
$$\eta = 1 - \frac{C_V (T_H - T_C)}{R T_H \log(V_2/V_1)}$$

(b)
$$\eta = 1 - \frac{T_C}{T_H}$$

(c)
$$\eta = 1 - \frac{C_V (T_H - T_C)}{C_P T_H}$$

(d)
$$\eta = 1 - \frac{C_V T_C}{R T_H \log(V_2/V_1)}$$

B7

Consider the function:

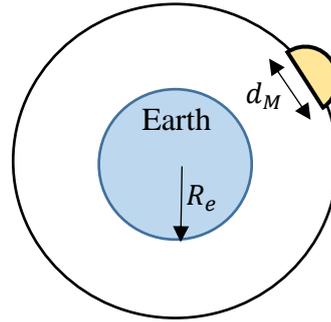
$$f(x) = \frac{4x - 11x^3}{(x^2 + 1)^2}$$

In the domain $x \in [1,4]$. What is the difference between the maximum value of the function and its minimum value? (Select the closest answer.)

- (a) 1.45
- (b) 3.2
- (c) 3.96
- (d) 0.82

B8

An earth imaging satellite is in a near earth circular orbit (see figure) with an orbital period of 100 minutes and observes earth with a light of wavelength 550 nm. If it can resolve the ground with 0.5 m, the minimum diameter of the mirror on the satellite should be closest to:

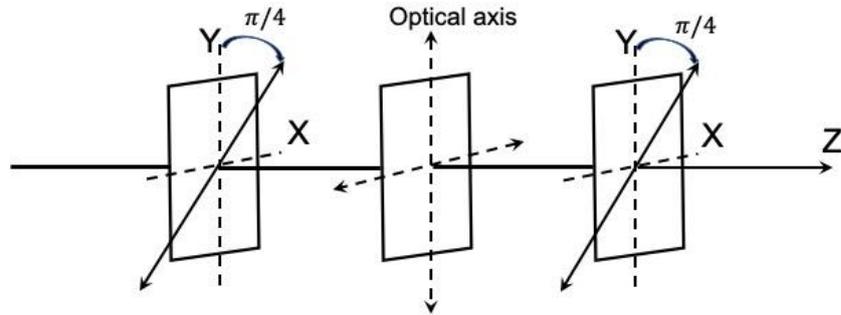


(Assume the system is diffraction limited. The mass and radius of Earth are 5.97×10^{24} kg and 6.4×10^3 km, respectively.)

(Figure not to scale)

- (a) 1 m
- (b) 5 m
- (c) 0.5 m
- (d) 10 m

- B9 In the experimental arrangement below, a plane polarised light of intensity I and wavelength 500 nm is propagating along the z -axis. It is polarised along the y -axis and is incident on the first polarizer which has the transmission axis at an angle of $\pi/4$ wrt the y -axis, as shown.



Subsequently, the beam passes through a flat birefringent crystal of thickness $950 \mu\text{m}$ and optical axis along the y -axis. The crystal is kept normal to the z -axis. If the refractive indices of the crystal are $n_o = 1.514$ and $n_e = 1.519$, the beam intensity measured after the second polarizer whose transmission axis is parallel to that of the first one would be:

- (a) 0
- (b) $I/4$
- (c) $I/2$
- (d) I

B10

The curve that solves $\frac{dy}{dx} = \frac{2y^3 - x^3}{3xy^2}$ and passes through the point (1,0) is given by:

(a) $y^3 + x^3 - x^2 = 0$

(b) $2y^3 - x^3 + x = 0$

(c) $y^3 + x^3 - x^2 - y^2 = 0$

(d) $2y^3 - x^3 + x + y = 0$

B11

The vector field, $E = (2xy^2 + z^3, 2x^2y, 3xz^2)$ V/m, gives the electric field in a region where (x, y, z) are the cartesian coordinates, given in metres. The difference in electric potential between the two points with coordinates $(x, y, z) = (1, 1, 1)$ and $(x, y, z) = (1, 2, 3)$, is given by:

- (a) -29 V
- (b) -16 V
- (c) 0 V
- (d) An electric potential cannot be defined for this electric field configuration

B12

Reactions in a closed container convert one molecule of A to one molecule of B at a rate Γ_1 and one molecule B to one molecule of C at a rate Γ_2 . At time $t = 0$, there are N_0 number of A molecules and no B or C molecules in the container. At what time is the number of B molecules in the container the maximum?

(a)
$$\frac{\log(\Gamma_2/\Gamma_1)}{\Gamma_2 - \Gamma_1}$$

(b)
$$\frac{\log(\Gamma_1/\Gamma_2)}{\Gamma_2 - \Gamma_1}$$

(c)
$$\frac{\log(\Gamma_2)}{\Gamma_2} + \frac{\log(\Gamma_1)}{\Gamma_1}$$

(d)
$$\frac{\log(\Gamma_2)}{\Gamma_2} - \frac{\log(\Gamma_1)}{\Gamma_1}$$

B13

Consider a quantum particle of mass m in one-dimension with a Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \lambda \hat{x}^8$$

with $\lambda > 0$. The ground state energy of the particle scales with λ as:

(a) $\propto \lambda^{1/5}$

(b) $\propto \lambda^{1/4}$

(c) $\propto \lambda^{1/3}$

(d) $\propto \lambda^{1/6}$

B14

A one-dimensional quantum mechanical wave function is given by:

$$\psi(x) = Ne^{-|x-b|/a},$$

where $x \in (-\infty, \infty)$ and N is a normalization factor. What is the expectation value of the kinetic energy in this state?

(a) $\frac{\hbar^2}{2ma^2}$

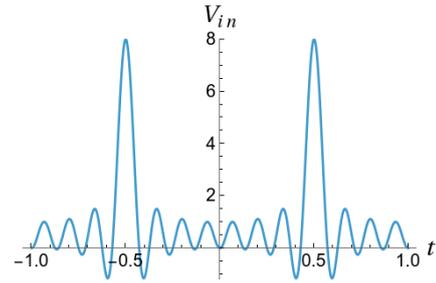
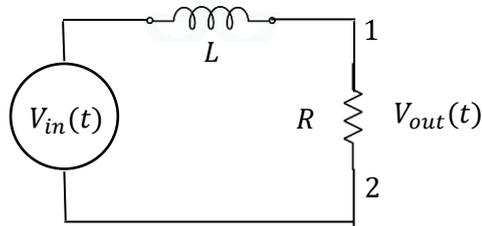
(b) $\frac{2\hbar^2}{ma^2}$

(c) $\frac{\hbar^2}{2ma^2} + \frac{\hbar^2}{2mb^2}$

(d) $\frac{2\hbar^2}{mb^2}$

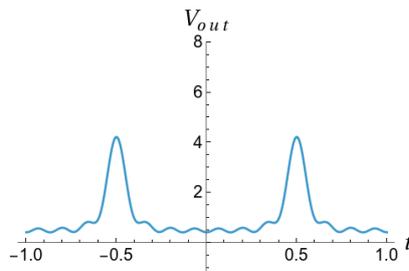
B15

The figure below shows a circuit. The input waveform $V_{in}(t)$ is shown. $R = 5\Omega$ and $L = 0.25 \text{ V}\cdot\text{s}/\text{A}$.

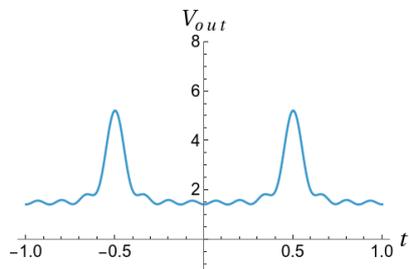


An ideal oscilloscope is connected between points 1 and 2, and shows a signal $V_{out}(t)$. What is the closest representation of $V_{out}(t)$?

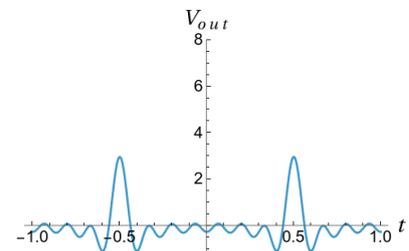
(a)



(b)



(c)



(d)

